Reg. No. :

Question Paper Code : 42768

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fourth Semester

Electronics and Communication Engineering

MA 2261 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulations 2008)

Time : Three Hours Maximum : 100 Marks

Use of Statistical Tables is Permitted

Answer ALL questions

PART – A (10×2=20 Marks)

1. If a random variable X has m.g.f Mx(t) = 3/3-t, then find the standard deviation of X.

2. If X and Y are independent random variable with variance 2 and 3, then find the variance of 3X + 4Y.

3. Let X and Y have the joint p.m.f.

Y \ X	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

Find P(X + Y > 1).

4. If Z = aX + bY and r is the correlation coefficient between X and Y, then show that $\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab r\sigma_x\sigma_y$.

5. For the sine wave process $X(t) = Y\cos\omega_0 t, -\infty<t<\infty$, ω_0 a constant the amplitude Y is a random variable with uniform distribution in the interval (0,1). Check whether the process is stationary or not.

6. Given that the auto correlation function for a stationary ergodic process with no periodic component is $R_{xx}(\tau) = 25 + 4/(1 + 6\tau^2)$. Find the mean value and variance of the process {X(t)}.

7. Check whether the function $1/1+4\tau^2$ is valid auto correlation functions.

8. Define power spectral density function of a stationary process.

9. Define a linear system.

10. Define a time invariant system.

PART – B (5×16=80 Marks)

11. a) 1) A continuous random variable X has p.d.f. $f(x) = kx^2e^{-x}, x \geq 0$. Find k, r^{th} raw moment, mean and variance. (8)

2) Let X be a random variable with uniform distribution in the interval (– a, a). Determine 'a' so that $P(-1 \leq X \leq 2) = 0.75$ and $P(|X| < 1) = P(|X| > 2)$. (8)

(OR)

b) 1) The amount of time that a watch can run without having to be reset is a random variable having exponential distribution with mean 120 days. Find the probability that such a watch will have to be reset in less than 24 days. Not have to be reset for at least 180 days. (8)

2) If X is the exponential distribution given by $f(x) = e^{-x}$ for $x > 0$ and zero otherwise, then find the probability density of $Y = \sqrt{X}$ and $Y = X^2$. (8)

12. a) 1) Find the correlation between X and Y, if the joint probability density of X and Y is $f(x, y) = 2$ for $x > 0, y > 0, x + y < 1$ and zero otherwise. (8)

2) Calculate the rank coefficient of correlation for the following data : (8)

X :	68	64	75	50	64	80	75	40	55	64
Y :	62	58	68	45	81	60	68	48	50	70

(OR)

b) 1) The joint p.d.f. of a two-dimensional random variable (X, Y) is given by $f(x, y) = 4xy\, e^{-(x^2+y^2)}$, $x \geq 0$ and $y \geq 0$ and zero otherwise. Find the probability density function of $U = (X^2 + Y^2)^{½}$. (8)

2) The joint p.d.f. of the R.V(X, Y) is given by $f(x, y) = x(1 + 3y^2)/4, 0<x<2$ and $0< y<1$. Find the marginal density function of X and Y, conditional density of X given Y and $P(1/4 < X < 1/2/Y = 1/3)$. (8)

13. a) 1) Consider a random process {X(t)} defined by X(t) = U cost + V sin t when U and V are independent random variable each of which assumes the values – 2 and 1 with probabilities 1/3 and 2/3 respectively. Show that {X(t)} is wide sense stationary and not strict sense stationary. (8)

2) Define random telegraph process. Prove that it is stationary in the wide sense. (8)

(OR)

b) 1) Suppose that X(t) is a random telegraph signal process with E[X(t)] = 0 and $R(\tau) = e^{-2\lambda|\tau|}$. Find mean and variance of the time average of X(t) over $(-\tau\ \tau)$. Is it mean ergodic ? (8)

2) Suppose X(t) is a normal process with mean $\mu(t) = 3$ and $C(t_1, t_2) = 4\, e^{-0.2}\, (|t_1 - t_2|)$. Find the probability that $X(5) \leq 2$ and $|X(8) - X(5)| \leq 1$. (8)

14. a) 1) If the process {X(t)} is defined as X(t) = Y(t) Z(t), where {Y(t)} and {Z(t)} are independent WSS process, prove that $R_{XX}(\tau) = R_{YY}(\tau)\, R_{ZZ}(\tau)$ and $S_{XX}(w) = (1/2\,\pi) \int_{-\infty}^{\infty} S_{YY}(\alpha) S_{ZZ}(w-\alpha)\, d\alpha$. (8)

2) Show that the spectral density function of a real random process is an even function. (8)

(OR)

b) 1) The power spectral density function of a zero mean WSS process {X(t)} is given by S(w) = 1, $|w| < w_0$ and zero, otherwise. Find $R(\tau)$. Show also that

$X(t)$ and $X\left(r + \frac{\tau}{w_0}\right)$ are uncorrelated. (8)

2) Define cross correlation function and write its 4 properties. (8)

15. a) If {X(t)} is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)\,(x(t-u))du$, then

$Ryy(\tau) = Rxx(\tau)*K(\tau)$, where $K(\tau)\ h(t)*\ h(-t) = \int_{-\infty}^{\infty} h(u)h((t+u))du$. (16)

(OR)

b) If the input to a time invariant stable linear system is a WSS process, then show that the output will also be a WSS process. (16)