Reg. No. : | | | | | | | | | | | | |

Question Paper Code : X 60768

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Fourth Semester
Electronics and Communication Engineering
MA 2261/MA 45/MA 1253/080380009/10177 PR 401 – PROBABILITY AND RANDOM PROCESSES
(Common to Biomedical Engineering)
(Regulations 2008/2010)

Time : Three Hours | Maximum : 100 Marks

(Use of Statistical tables is permitted)

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. The cumulative distribution function of the random variable X is given by
$$F_X(x) = \begin{cases} 0\,; & x < 0 \\ x + \frac{1}{2}\,; & 0 \le x \le \frac{1}{2} \\ 1; & x > \frac{1}{2} \end{cases}, \text{ compute } P\left[X > \frac{1}{4}\right].$$

2. Find the variance of the discrete random variable X with the probability mass function $P_X(x) = \begin{cases} \frac{1}{3}, & x = 0 \\ \frac{1}{2}, & x = 2 \end{cases}$.

3. The joint pdf of (X, Y) is given by $f(x, y) = k\, xy e^{-(x^2 + y^2)}$; $x > 0$, $y > 0$. Find the value of k.

4. Define the distribution function of two dimensional random variable (X, Y). State any one property.

5. Define a strictly stationary random process.

6. Prove that sum of two independent Poisson processes is again a Poisson process.

7. A random process X(t) is defined by $X(t) = K \cos\omega t$, $t \geq 0$ where ω is a constant and K is uniformly distributed over (0, 2). Find the autocorrelation function of X(t).

8. Define cross correlation function of X(t) and Y(t). When do you say that they are independent ?

9. Define white noise process.

10. Define linear time invariant system.

PART – B **(5×16=80 Marks)**

11. a) i) A random variable X has pdf $f_x(x) = \begin{cases} kx^2 e^{-x} ; & x > 0 \\ 0 & \text{otherwise} \end{cases}$. Find the r^{th} moment of X about origin. Hence find the mean and variance. **(8)**

ii) A random variable X is uniformly distributed over (0, 10). Find

1) $P(X < 3)$, $P(X > 7)$ and $P(2 < X < 5)$

2) $P(X = 7)$. **(8)**

(OR)

b) i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act independently.

1) What is the probability that all four phones are busy ?

2) What is the probability that atleast two of them are busy ? **(6)**

ii) Describe gamma distribution. Obtain its moment generating function. Hence, compute its mean and variance. **(10)**

12. a) i) State and prove central limit theorem for independently and identically distributed (iid) random variables. **(6)**

ii) If X and Y are independent RVs with pdf's $e^{-x}; x \geq 0$ and $e^{-y}; y \geq 0$, respectively, find the pdfs of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V independent ? **(10)**

(OR)

b) The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of (X + Y).

13. a) i) Examine whether $X(t) = A \cos\lambda t + B \sin\lambda t$ where A and B are random variables such that $E(A) = E(B) = 0$; $E(A^2) = E(B^2)$; $E(AB) = 0$, is wide sense stationary. **(8)**

ii) Find the auto correlation function of the Poisson process. **(8)**

(OR)

b) i) Suppose X(t) is a normal process with mean $\mu(t) = 3$, $C_x(t_1, t_2) = 4e^{-0.2|t_1 - t_2|}$. Find $P(X(5) \le 2)$ and $P(|X(8) - X(5)| \le 1)$. **(8)**

ii) Define a random telegraph process. Show that it is a covariance stationary process. **(8)**

14\. a) i) Find the spectral density of a WSS random process {X(t)} whose autocorrelation function is $e^{\frac{-\alpha^2 t^2}{2}}$. **(8)**

ii) Find the autocorrelation function of the WSS process {X(t)} whose spectral density is given by $S(\omega) = \dfrac{1}{(1+\omega^2)^2}$. **(8)**

(OR)

b) i) The cross-power spectrum of real random process {X(t)} and {Y(t)} is given by $S_{XY}(\omega) = \begin{cases} a + jb\omega, & |\omega| < 1 \\ 0 & \text{elsewhere} \end{cases}$. Find the cross-correlation function. **(8)**

ii) Determine the cross correlation function corresponding to the cross-power density spectrum $S_{XY}(\omega) = \dfrac{8}{(\alpha + j\omega)^3}$, where $\alpha > 0$ is a constant. **(8)**

15\. a) i) Show that if the input {X(t)} is a WSS process for a linear system then output {Y(t)} is a WSS process. Also find $R_{XY}(\tau)$. **(8)**

ii) If X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \dfrac{1}{\omega + 2i}$. **(8)**

(OR)

b) i) If $Y(t) = A\cos(\omega_0 t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and {N(t)} is a band-limited Gaussian white noise with power spectral density $S_{NN}(\omega) = \begin{cases} \dfrac{N_0}{2}, & \text{for } |\omega - \omega_0| < \omega_B \\ 0, & \text{elsewhere} \end{cases}$.

Find the power spectral density {Y(t)}. Assume that {N(t)} and θ are independent. **(10)**

ii) A system has an impulse response $h(t) = e^{-\beta t}\, U(t)$, find the power spectral density of the output Y(t) corresponding to the input X(t). **(6)**
