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Question Paper Code : 80766

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/080380009/10177 PR 401 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulations 2008/2010)

Time : Three hours Maximum : 100 marks

(Use of Statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If a random variable X takes values 1, 2, 3, 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$. Find the probability distribution of X.

2. Find the moment generating function of Poisson distribution.

3. The joint pdf of $RV\,(x,y)$ is given by $f(x,y)=k\ \ xye^{-(x^2+y^2)}$; $x>0,\ y>0$. Find the value of k.

4. Given the $R \vee X$ with density function

   $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & elsewhere\end{cases}$ Find the pdf of $y=8x^3$.

5. Define first-order stationary processes.

6. Suppose that $X(t)$ is a Gaussian process with $\mu_x=2, R_{XX}=(\tau)=5e^{-0.2|\tau|}$, find the probability that $X(4)\le 1$.

7. Find the variance of the stationary process $\{X(t)\}$ whose auto correlation function is given by $R_{XX}(\tau) = 2 + 4e^{-2|\tau|}$.

8. Prove that for a WSS process $\{X(t)\}, R_{XX}(t, t+\tau)$ is an even function of τ.

9. Define a linear time invariant system.

10. State the convolution form of the output of a linear time invariant system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The members of a girl scout troop are selling cookies from house to house in town. The probability that they sell a set of cookies at any house they visit is 0.4.

(1) If they visit 8 houses in one evening, what is the probability that they sold cookies to exactly five of these houses?

(2) If they visited 8 houses in one evening, what is the expected number of sets of cookies they sold?

(3) What is the probability that they sold their set of cookies atmost in the sixth house they visited? (8)

(ii) Suppose X has an exponential distribution with mean equal to 10. Find the value of x such that $P(x < x) = 0.95$. (8)

Or

(b) (i) If the moments of a random variable X are defined by $E(X^r) = 0.6$, $r = 1,2...$ Show that $P(X = 0) = 0.4$, $P(X = 1) = 0.6$ and $P(X \geq 2) = 0$. (8)

(ii) Find the probability density function of the random variable $y = x^2$ where X is the standard normal variate. (8)

12. (a) (i) State and prove central limit theorem for iid RVs. (8)

(ii) If X and Y are independent RVs with pdfs $e^{-x}; x \geq 0$ and e^{-y}; $y \geq 0$, respectively, find the pdfs of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V independent? (8)

Or

(b) The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y)$, $x = 0,1,2;$ $y = 1,2,3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of $(X + Y)$. (16)

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t)=n\}=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}} & n=1,2..\\ \dfrac{at}{1+at}, & n=0\end{cases}$$

Find the mean and variance of the process. Is the process first-order stationary? (8)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t)=10\cos(100t+\theta)$, where θ is uniformly distributed over $(-\pi,\pi)$, prove that $\{X(t)\}$ is correlation ergodic. (8)

Or

(b) (i) If the process $\{X(t);t\geq 0\}$ is a Poisson process with parameter λ, obtain $P[X(t)=n]$. Is the process first order stationary? (8)

(ii) Prove that a random telegraph signal process $Y(t)=\alpha X(t)$ is a Wide Sense Stationary Process when α is a random variable which is independent of $X(t)$, assumes values -1 and $+1$ with equal probability and $R_{XX}(t_1,t_2)=e^{-2\lambda|t_1-t_2|}$. (8)

14. (a) (i) Define spectral density of a stationary random process $X(t)$. Prove that for a real random process $X(t)$ the power spectral density is an even function. (8)

(ii) Two random processes $X(t)$ and $Y(t)$ are defined as follows:

$X(t)=A\cos(wt+\theta)$ and $Y(t)=B\sin(wt+\theta)$ where A,B and w are constants; θ is a uniform random variable over $(0,2\pi)$. Find the cross correlation function of $X(t)$ and $Y(t)$. (8)

Or

(b) (i) State and prove Wiener — Khintchine theorem. (8)

(ii) If the cross power spectral density of $X(t)$ and $Y(t)$ is

$$S_{XY}(w)=\begin{cases}a+\dfrac{ibw}{\theta}; -\alpha<w<\alpha, \alpha>0\\ 0 \qquad otherwise\end{cases}$$ where a and b are constants.

Find the cross correlation function. (8)

15. (a) (i) Prove that if the input to a time-invariant stable linear system is a wide sense process then the output also is a wide sense process. (8)

(ii) A random process $X(t)$ with $R_{XX}(\tau) = e^{-2|\tau|}$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t > 0$. Find the cross correlation coefficient $R_{XY}(\tau)$ between the input process $X(t)$ and output process $Y(t)$. (8)

Or

(b) (i) Let $X(t)$ be a wide sense stationary process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$. Prove that $S_{YY}(w) = |H(w)|^2 S_{XX}(w)$ where $H(w)$ is the Fourier transform of $h(t)$. (8)

(ii) Let $Y(t) = X(t) + N(t)$ be a wide sense stationary process where $X(t)$ is the actual signal and $N(t)$ is the zero mean noise process with variance σ_N^2, and independent of $X(t)$. Find the power spectral density of $Y(t)$. (8)