Reg. No. :

Question Paper Code : 41312

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fourth Semester

Electronics and Communication Engineering

MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions

PART – A

(10×2=20 Marks)

1. If $f(x) = \frac{x^2}{3}, -1 < x < 2$ is the pdf of the random variable X, then find $P[0 < x < 1]$.
2. Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for exactly two messages arrive within one hour.
3. If X and Y are random variables having the joint density function $f(x, y) = \frac{1}{8}(6 - x - y)$, $0 < x < 2,\ 2 < y < 4$, then find $P[X + Y < 3]$.
4. Find the acute angle between the two lines of regression.
5. Define Markov process.
6. State any two properties of Poisson process.
7. Find the mean square value of the random process {X(t)} if its autocorrelation function $R(\tau) = 25 + \frac{4}{1 + 6\tau^2}$.
8. Write any two properties of the power spectral density of the WSS process.
9. Prove that the mean of the output process is the convolution of the mean of the input process and the impulse response.
10. Assume that the input X(t) to a linear time-invariant system is white noise. What is the power spectral density of the output process Y(t) if the system response $H(\omega) = 1, \omega_1 < |\omega| < \omega_2$ is given ?
$= 0$, otherwise

PART – B (5×16=80 Marks)

11. a) i) For a uniform random variable X in the interval (a, b), derive the moment generating function and hence obtain its mean and variance. (10)

ii) Let X be the random variable that denotes the outcome of the roll of a fair die. Compute the mean and variance of X. (6)

(OR)

b) i) Find the moment generating function of Gamma distribution with parameters K and λ and hence compute the first four moments. (10)

ii) A continuous random variable X has the density function f(x) given by $f(x) = \frac{k}{x^2+1}, -\infty < x < \infty$. Find the value of k and the cumulative distribution of X. (6)

12. a) Given $f(x, y) = \frac{1}{8}(x + y), 0 \le x \le 2, 0 \le y \le 2$ is the joint pdf of X and Y. Obtain the correlation coefficient between X and Y.

(OR)

b) i) Let (X, Y) be a two dimensional non-negative continuous random variable having the joint probability density function $f(x, y) = 4xy\, e^{-(x^2+y^2)}, x \ge 0, y \ge 0$. Find the probability density function of $\sqrt{X^2 + Y^2}$. (10)

ii) Find P[X < Y/X < 2Y] if the joint pdf of (X, Y) is $f(x, y) = e^{-(x+y)}, 0 \le x < \infty, 0 \le y < \infty$. (6)

13. a) i) Prove that Poisson process is a Markov process. (8)

ii) A random process {X(t)} is defined by X(t) = A cost + B sintt, $-\infty < t < \infty$, where A and B are independent random variables each of which has a value –2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. Show that {X(t)} is not stationary in strict sense. (8)

(OR)

b) i) If $\{X_1(t)\}$ and $\{X_2(t)\}$ represent two independent Poisson processes with parameters $\lambda_1 t$ and $\lambda_2 t$ respectively, then prove that $P[X_1(t) = x/\{X_1(t) + X_2(t) = n\}]$ is binomial with parameters n and p, where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$. (10)

ii) Consider a random process {X(t)} such that $X(t) = A\cos(\omega t + \theta)$, where A and ω are constants, and θ is a uniform random variable distributed with interval $(-\pi, \pi)$. Check whether the process {X(t)} is a stationary process in wide sense. (6)

14. a) i) Consider two random processes $X(t) = 3\cos(\omega t + \theta)$ and $Y(t) = 2\cos\left(\omega t + \theta - \frac{\pi}{2}\right)$, where θ is a random variable uniformly distributed in $(0, 2\pi)$. Prove that $\sqrt{R_{XX}(0)\, R_{YY}(0)} \ge |R_{XY}(\tau)|$. (10)

ii) Determine the autocorrelation function of the random process with the power spectral density given by $S_{XX}(\omega) = S_0$, $|\omega| < \omega_0$. (6)

(OR)

b) i) Given that a process {X(t)} has the autocorrelation function $R_{XX}(\tau) = Ae^{-\alpha-|\tau|}\cos(\omega_0 \tau)$ where A > 0, $\alpha > 0$ and ω_0 are real constants, find the power spectrum of X(t). (8)

ii) The cross-power spectrum of real random processes X(t) and Y(t) is given by $S_{XY}(\omega) = a + jb\omega$, $|\omega| < 1$. Find the cross-correlation function. (8)

15. a) i) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$, where $S_{XX}(\omega)$ and $S_{YY}(\omega)$ are the power spectral density functions of the input X(t) and the output Y(t) and $H(\omega)$ is the system transfer function. (8)

ii) Obtain the power spectral density function of the output process {Y(t)} corresponding to the input process {X(t)} is the system that has an impulse response $h(t) = e^{-\beta t} U(t)$. (8)

(OR)

b) A random process X(t) is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t \ge 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, determine the following :

The cross correlation function between the input process X(t) and the output process Y(t).