Reg. No. :

Question Paper Code : 91784

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Fourth Semester

Electronics and Communication Engineering

MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Prove that the function p(x) is a legitimate probability mass function of a discrete random variable X, where $p(x) = \begin{cases} \frac{2}{3}\left(\frac{1}{3}\right)^x; & x = 0, 1, 2, \ldots \\ 0; & \text{otherwise} \end{cases}$

2. Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box.

3. Let (X, Y) be a two-dimensional random variable, define covariance of (X, Y). If X and Y are independent, what will be the covariance of (X, Y) ?

4. If the joint pdf of (X, Y) is $f(x, y) = \begin{cases} \frac{1}{x}, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$. Find the marginal pdf of Y.

5. Give an example of evolutionary random process.

6. Define a semi-random telegraph signal process.

7. Prove that the auto correlation function is an even function of τ.

8. Compute the mean value of the random process {X(t)} whose autocorrelation function is given by $R(\tau) = 25 + 4/(1+6\tau^2)$.

9. Define casual system.

10. Define transfer function of a system.

PART – B (5×16=80 Marks)

11. a) i) A continuous random variable X that can assume any value between X = 2 and X = 5 has a probability density function given by f(x) = k(1 + x). Find P(X < 4). (8)

ii) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test on the 4th trial ? Also find the probability that he will finally pass the test in less than 4 trials. (8)

(OR)

b) i) Find the moment generating function of exponential distribution and hence find the mean and variance of exponential distribution. (8)

ii) If the probability mass function of a random variable X is given by $P[X = x] = kx^3$, x = 1, 2, 3, 4, find the value of k, $P\left[\left(\frac{1}{2} < X < \frac{5}{2}\right) \Big/ X > 1\right]$, mean and variance of X. (8)

12. a) i) The joint probability mass function of (X, Y) is given by $p(x, y) = \frac{1}{72}(2x + 3y)$, x = 0, 1, 2 and y = 1, 2, 3. Find all the marginal and conditional probability functions of X and Y. (10)

ii) The joint pdf of (X, Y) is $f(x, y) = e^{-(x+y)}$, $x, y \geq 0$. Are X and Y independent ? (6)

(OR)

b) i) The joint pdf of a random variable (X, Y) is $f(x, y) = 25e^{-5y}$ $0 < x < 0.2$, $y > 0$. Find the covariance of X and Y. (8)

ii) The random variables X and Y each follow exponential distribution with parameter 1 and are independent. Find the pdf of U = X – Y. (8)

13. a) i) Show that the random process $X(t) = A \cos(wt + \theta)$ is wide-sense stationary, where A and w are constants and θ is uniformly distributed on the interval (0, 2π). (8)

ii) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the mass tossed a fair die and drove to work if and only if 6 appeared. Find 1) the probability that he takes a train on the third day and 2) the probability that he drives to work in the long run. (8)

(OR)

b) i) Prove that the difference of two independent Poisson processes is not a Poisson process. (8)

ii) Prove that a random telegraph signal process $y(t) = \alpha\, x(t)$ is a WSS process, where α is a random variable which is independent of x(t), assumes values –1 and 1 with equal probability and $R_{xx}(t_1, t_2) = e^{-2\lambda|t_1 - t_2|}$. (8)

14. a) i) The cross-power spectrum of real random process {X(t)} and {Y(t)} is given by $S_{XY}(\omega) = \begin{cases} a + jb\omega, & |\omega| < 1 \\ 0 & \text{elsewhere} \end{cases}$. Find the cross correlation function. (8)

ii) Find the power spectral density of the random process if its autocorrelation function is $R(\tau) = e^{-\alpha\tau^2} \cos \omega_0\tau$. (8)

(OR)

b) i) Find the mean, variance and Root-mean square value of the process whose auto correlation function is $R_{XX}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$. (8)

ii) Find the autocorrelation of the process {X(t)} which the spectral density is given by $S(\omega) = \begin{cases} 1 + \omega^2, & |\omega| \leq 1 \\ 0, & |\omega| > 1 \end{cases}$. (8)

15. a) i) Let X(t) be the input voltage to a circuit and Y(t) be the output voltage. {X(t)} is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find μ_Y, $S_{YY}(\omega)$ if the power transfer function is $H(\omega) = \frac{R}{R + iL\omega}$. (8)

ii) If a system is such that its input X(t) and its output Y(t) are related by a convolution integral, prove that the system is a linear time invariant system. (8)

(OR)

b) A random process X(t) is the input to a linear system whose impulse response is given by $h(t) = 2e^{-t}$, $t \geq 0$. If the autocorrelation function of the process X(t) is $R_{XX}(\tau) = e^{-2|\tau|}$, determine the cross correlation function $R_{XY}(\tau)$ and $R_{YX}(\tau)$ between the input process X(t) and the output process Y(t). (16)