Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 23768

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third/Fifth Semester

Civil Engineering

MA 2211 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulations 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $f(x) = x^2$ in $-2 < x < 2$ and $f(x+4) = f(x)$, then find the coefficient a_0 of its Fourier series.

2. Find the root mean square value of $f(x) = \cos x$ in $(0, 2\pi)$.

3. State the convolution theorem for Fourier transform.

4. Show that the Fourier transform satisfies the linearity property.

5. Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$.

6. Find the complete integral of $p = 2qx$.

7. Mathematically formulate the following vibrating string problem: "A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given an initial velocity $f(x)$, x being the distance from an end point".

8. Classify the partial differential equation : $u_{xx} + 2u_{xy} + u_{yy} = 0$.

9. Find the Z transform of $u(n-1)$.

10. State the initial value theorem of Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a Fourier series to represent $f(x) = x - x^2$ from $-\pi$ to π. (10)

(ii) Obtain the constant term a_0 and the first two harmonics a_1 and a_2 in the Fourier cosine series representation of $y = f(x)$ in (0, 6) for the given table of values: (6)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Or

(b) (i) Find the half range Fourier sine series expansion for

$$f(x) = \begin{cases} \frac{2kx}{L}, & 0 < x < L/2 \\ \frac{2k(L-x)}{L}, & L/2 < x < L \end{cases}$$. Hence evaluate $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$. (8)

(ii) Find the complex form of the Fourier series for $f(x) = e^{-x}, -1 < x < 1$. (8)

12. (a) (i) Express the function $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier integral.

Hence evaluate $\int_0^{\infty} \frac{\sin \lambda}{\lambda} d\lambda$. (10)

(ii) Find the Fourier sine transform of $\frac{1}{x}$. (6)

Or

(b) (i) Find the Fourier transform of $e^{-a^2x^2}, a > 0$ and hence find the Fourier transform of $e^{-x^2/2}$. (8)

(ii) Using Fourier transform methods, evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$, $a, b > 0$. (8)

13. (a) (i) Solve the partial differential equation: $z^2(p^2 + q^2) = x^2 + y^2$. (8)

(ii) Solve $(D^2 + 2DD' + D'^2)z = \sinh(x + y) + e^{x+2y}$. (8)

Or

(b) (i) Solve $(3z - 4y)p + (4x - 2z)q = 2y - 3x$. (8)

(ii) Obtain the complete solution and singular solution of $z = px + qy + p^2q^2$. (8)

14. (a) A rod, 30 cm long, has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function $u(x, t)$ taking $x = 0$ at A. (16)

Or

(b) An infinitely long plane uniform plate is bounded by two parallel edges $x = 0$ and $x = l$ and an edge at right angles to them. The breadth of this edge $y = 0$ is l and is maintained at a temperature $f(x)$. All the other three edges are at zero temperature. Find the steady state temperature at any interior point of the plate. (16)

15. (a) (i) Using Z-transform method, solve the difference equation: $x(n+1) - 2x(n) = 1$ given $x(0) = 0$. (10)

(ii) Find the Z transform of t^k and hence deduce the result: $Z(t^k) = -Tz\frac{d}{dz}\{Z(t^{k-1})\}$, where T is the sampling period with $t = nT, n = 0, 1, 2, \ldots$ (6)

Or

(b) (i) Find the inverse Z transform of $\frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}$ using partial fractions method. (8)

(ii) Find the inverse Z transform $\frac{z^2}{(z-a)^2}$ using convolution theorem. (8)