Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 31264

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third Semester

Civil Engineering

MA 2211 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all Branches)

(Regulation 2008)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the root mean square value of $f(x) = x - x^2$ in the interval $-1 < x < 1$.

2. Obtain the complex form of Fourier series for the function

   $$f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ 1, & 0 \le x \le \pi \end{cases}$$

3. Find the Fourier cosine transform of $f(x) = 3e^{-2x} + 2e^{-3x}$.

4. State the convolution theorem for Fourier transform.

5. Form the differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.

6. Find the complete solution of $z = px + qy + p^2q^2$.

7. Classify the PDE given by $f_{xx} + 2f_{xy} + 4f_{yy} = 0$.

8. Write all the possible solutions of the one dimensional heat equation $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial y^2}$.

9. Find $Z\left(\dfrac{a^n}{n!}\right)$.

10. State the final value theorem of Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that for $0 < x < \pi$

$$x(\pi - x) = \frac{8}{\pi}\left\{\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \ldots\right\}.$$ Using Parseval's identity, show that $\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$. (8)

(ii) Obtain the constant term and the coefficient of first sine and cosine term in the Fourier expansion of $y = f(x)$ as given in the following table. (8)

x:	0	1	2	3	4	5
$f(x)$:	9	18	24	28	26	20

Or

(b) Expand $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi \end{cases}$ as a Fourier series and hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots$. (16)

12. (a) (i) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1. \end{cases}$$

Hence evaluate $\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2}\, dx$. (8)

(ii) Using Parseval's identities, prove that

$$\int_0^{\infty} \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}.$$ (8)

Or

(b) (i) Find the Fourier cosine transform of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2. \\ 0, & x > 2 \end{cases}$$ (8)

(ii) Solve the integral equation

$$\int_0^{\infty} f(x) \sin tx\, dx = \begin{cases} 1, & 0 < t < 1 \\ 2, & 1 < t < 2. \\ 0, & t > 2 \end{cases}$$ (8)

13. (a) (i) Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$. (8)

(ii) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$. (8)

Or

(b) (i) Find the complete solution of $z^2(p^2 + q^2) = x^2 + y^2$. (8)

(ii) Solve $(D^3 - 7DD'^2 - 6D'^3)\, z = e^{2x+y}$. (8)

14. (a) A tightly streached string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $F(x) = \mu x(l - x)$ where μ is a constant and then released. Find the displacement at any point x of the string at any time $t > 0$. (16)

Or

(b) A bar 10 cm long with insulated sides has its ends A and B maintained at temperatures 50°C and 100°C respectively until steady state prevails. The temperature at A is suddenly raised to 90°C and at the same time the temperature at B is lowered to 60°C. Find the temperature distribution of the bar at time t. (16)

15. (a) (i) If $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, then evaluate u_2 and u_3. (8)

(ii) Find $Z^{-1}\left(\frac{z^2}{(z-a)(z-b)}\right)$ using convolution theorem. (8)

Or

(b) (i) Find the inverse Z transform of $\frac{5z}{(2-z)(3z-1)}$ using partial fraction method. (8)

(ii) Solve $y_{n+2} - 2y_{n+1} + y_n = 2^n$ with $y_0 = 2$ and $y_1 = 1$ using Z transform. (8)