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**Question Paper Code : 50779**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Third Semester

Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Mechanical Engineering (Sandwich)/ Aeronautical Engineering/  
Agriculture Engineering/ Automobile Engineering/ Biomedical Engineering/  
Computer Science and Engineering/ Electrical and Electronics Engineering/  
Electronics and Communication Engineering/ Electronics and Instrumentation  
Engineering/ Geoinformatics Engineering/ Industrial Engineering/ Industrial  
Engineering and Management/ Instrumentation and Control Engineering/  
Manufacturing Engineering/ Marine Engineering/ Materials Science and  
Engineering/Mechanical Engineering/Mechanical and Automation Engineering/  
Mechatronics Engineering/ Medical Electronics/ Petrochemical Engineering/  
Production Engineering/ Robotics and Automation Engineering/ Biotechnology,  
Chemical Engineering/ Chemical and Electrochemical Engineering/  
Food Technology/ Information Technology/ Petrochemical Technology/ Petroleum  
Engineering/ Plastic Technology/Polymer Technology)  
(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Find the partial differential equation by eliminating the arbitrary function 'f' from the relation  $z = f(x^2 - y^2)$ .
2. Find the complete integral of  $\sqrt{p} + \sqrt{q} = 1$ .
3. State Dirichlet's conditions for a given function  $f(x)$  to be expanded in Fourier series.
4. Write the complex form of Fourier series for a function  $f(x)$  defined in  $-l < x < l$ .

5. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation ?
6. State any two solutions of the Laplace equation  $u_{xx} + u_{yy} = 0$  involving exponential terms in  $x$  or  $y$ .
7. If  $F[f(x)] = F(s)$ , then find  $F[f(ax)]$ .
8. State the convolution theorem for Fourier transforms.
9. Find the Z-transform of the function  $f(n) = 1/n$ .
10. Form the difference equation by eliminating arbitrary constant 'a' from  $y_n = a \cdot 2^n$ .

## PART - B

(5×16=80 Marks)

11. a) i) Find the singular integral of  $z = px + qy + p^2 - q^2$ . (8)
- ii) Find the general integral of  $(x - 2z)p + (2z - y)q = y - x$ . (8)
- (OR)
- b) Solve the following equations.
- i)  $(D^2 + 2DD' + D'^2)z = e^{x-y} + xy$  (8)
- ii)  $(D^2 - 5DD' + 6D'^2)z = y \sin x$ . (8)
12. a) i) Find the Fourier series for a function  $f(x) = x + x^2$  in  $(-\pi, \pi)$  and hence deduce the value of  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  (8)

- ii) Find the Fourier series of  $y = f(x)$  up to first harmonic which is defined by the following data in  $(0, 2\pi)$ :

x	0	$\pi/3$	$2\pi/3$	$\pi$	$4\pi/3$	$5\pi/3$	$2\pi$
f(x)	1	1.4	1.9	1.7	1.5	1.2	1

(OR)

- b) i) Find the half-range cosine series for  $f(x) = x$  in  $(0, \pi)$ . Hence deduce the value of  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$  (8)
- ii) Find the Fourier series for a function  $f(x) = \begin{cases} l-x, & 0 < x \leq l \\ 0, & l < x \leq 2l \end{cases}$  in  $(0, 2l)$ . (8)

13. a) A tightly stretched string of length  $l$  has its end fastened at  $x = 0, x = l$ . At  $t = 0$ , the string is in the form  $f(x) = kx(l-x)$  and then released. Find the displacement at any point of the string at a distance  $x$  from one end and at any time  $t > 0$ . (16)
- (OR)

- b) A rod of length  $l$  cm has its ends A and B kept at  $0^\circ\text{C}$  and  $100^\circ\text{C}$  respectively, until steady state conditions prevail. If the temperature at B is suddenly reduced to  $0^\circ\text{C}$  and maintained at  $0^\circ\text{C}$ , find the temperature distribution  $u(x, t)$  at a distance  $x$  from A at any time  $t$ . (16)

14. a) i) If  $F_S(s)$  and  $F_C(s)$  denote Fourier sine and cosine transform of a function  $f(x)$  respectively, then show that

$$F_S\{f(x) \sin ax\} = \frac{1}{2} \{F_C(s-a) - F_C(s+a)\} \quad (4)$$

- ii) Find the Fourier transform of a function  $f(x) = \begin{cases} 1-|x| & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$  and hence

$$\text{find the value of } \int_0^\infty \frac{\sin^4 t}{t^4} dt \text{ by Parseval's identity.} \quad (12)$$

(OR)

- b) Find the Fourier sine and cosine transforms of a function  $f(x) = e^{-x}$ . Using Parseval's identity, evaluate:

$$(1) \int_0^\infty \frac{dx}{(x^2+1)^2} \text{ and } (2) \int_0^\infty \frac{x^2 dx}{(x^2+1)^2} \quad (16)$$

15. a) i) Find the Z-transform of  $\frac{2n+3}{(n+1)(n+2)}$ . (8)

ii) Find  $Z^{-1} \left[ \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)} \right]$  by using convolution theorem. (8)

(OR)

- b) i) Find the inverse Z-transform of  $\frac{z^3}{(z-1)^2(z-2)}$  by method of partial fraction. (6)

- ii) Solve the difference equation  $y(n+2) - 7y(n+1) + 12y(n) = 2^n$ , given that  $y(0) = 0$  and  $y(1) = 0$ , by using Z-transform. (10)