Reg. No. :

Question Paper Code : X20781

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020 AND APRIL/MAY 2021

Third Semester

Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all Branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulations 2013)

(Also common to PTMA 6351 – Transforms and Partial Differential Equations for B.E. Part-time – Civil Engineering, Electronics and Communication Engineering, Mechanical Engineering – Second Semester – Regulations 2014)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Construct the partial differential equation of all spheres whose centres lie on the z-axis, by the elimination of arbitrary constants.

2. Solve $(D + D' - 1)(D - 2D' + 3)z = 0$.

3. The instantaneous current 'i' at time t of an alternating current wave is given by $i = I_1 \sin(\omega t + \alpha_1) + I_3 \sin(3\omega t + \alpha_3) + I_5 \sin(5\omega t + \alpha_5) + \dots$. Find the effective value of the current 'i'.

4. If the Fourier series of the function $f(x) = x + x^2$, in the interval $(-\pi, \pi)$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty}(-1)^n\left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right]$, then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

5. State the assumptions in deriving one-dimensional wave equation.

6. State the three possible solutions of the one-dimensional heat (flow unsteady state) equation.

7. If F(s) is the Fourier transform of f(x), prove that $F\{f(x-a)\} = e^{ias}F(s)$.

8. Find Fourier sine transform of $\frac{1}{x}$

9. Find $Z\left[\cos\left(\frac{n\pi}{2}\right)\right]$.

10. State initial and final value theorem for Z-transforms.

PART – B **(5×16=80 Marks)**

11. a) i) Solve : $(x^2 - yz)\,p + (y^2 - xz)q = (z^2 - xy)$. **(8)**

ii) Solve : $(D^2 - 3DD' + 2D'^2)z = (2+4x)e^{x+2y}$. **(8)**

(OR)

b) i) Obtain the complete solution of $p^2 + x^2y^2q^2 = x^2z^2$. **(8)**

ii) Solve $z = px + qy + p^2q^2$ and obtain its singular solution. **(8)**

12. a) i) Expand $f(x) = x^2$ as a Fourier series in the interval $(-\pi, \pi)$ and hence deduce that $1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + ... = \frac{\pi^4}{90}$. **(8)**

ii) Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of y as given in the following table : **(8)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(OR)

b) i) Expand $f(x) = e^{-ax}$, $-\pi < x < \pi$ as a complex form Fourier series. **(8)**

ii) Expand $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$ as a series of cosines in the interval (0, 2). **(8)**

13. a) A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $v = \begin{cases} \frac{2kx}{l} & \text{in } 0 < x < \frac{l}{2} \\ \frac{2k(l-x)}{l} & \text{in } \frac{1}{2} < x < l \end{cases}$. Find the displacement of the string at any distance x from one end at any time t. **(16)**

(OR)

b) A bar 10 cm long with insulated sides has its ends A and B maintained at temperature 50°C and 100°C, respectively, until steady state conditions prevails. The temperature at A is suddenly raised to 90°C and at the same time lowered to 60°C at B. Find the temperature distributed in the bar at time t. **(16)**

14. a) Find the Fourier transform of f(x) given by $f(x)=\begin{cases}1 & \text{for } |x|<2\\ 0 & \text{for } |x|>2\end{cases}$ and hence evaluate $\int_0^{\infty}\frac{\sin x}{x}\,dx$ and $\int_0^{\infty}\left(\frac{\sin x}{x}\right)^2 dx$. **(16)**

(OR)

b) i) Find the Fourier cosine transform of $e^{-a^2x^2}$ for any $a > 0$. **(8)**

ii) Evaluate $\int_0^{\infty}\frac{dx}{(x^2+1)(x^2+4)}$ using Fourier transforms. **(8)**

15. a) i) Find $Z\left[\frac{1}{(n+1)(n+2)}\right]$. **(8)**

ii) Using convolution theorem evaluate $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$. **(8)**

(OR)

b) i) Using Z-Transform solve $y(n+3) - 3y(n+1) + 2y(n) = 0$, with $y(0) = 4$, $y(1) = 0$, $y(2) = 8$. **(8)**

ii) Find $Z^{-1}\left[\frac{z}{(z-1)(z^2+1)}\right]$ by using integral method. **(8)**
