Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 70768

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/Agriculture Engineering/Automobile Engineering/Biomedical Engineering/Computer Science and Engineering/Computer and Communication Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Geoinformatics Engineering/Industrial Engineering/ Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Information Technology/Petrochemical Technology/Petroleum Engineering/Plastic Technology/Polymer Technology)

(Regulation 2013)

(Also common to : PTMA 6351 – Transforms and Partial Differential Equations for B.E. (Part-Time) – Civil Engg./ Electronics and Communication Engg., – Mechanical Engg., – Second Semester – (Regulations – 2014))

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from $\log(az-1)\, x + ay + b$.

2. Find the complete solution of $q = 2px$.

3. State Dirichlet condition for a given function $f(x)$ to be expanded in Fourier series.

4. Write the complex form of Fourier series for a function $f(x)$ defined in $-l < x < l$.

5. Solve $yu_x + xu_y = 0$ using separation of variable method.

6. What are the possible solutions of the one dimensional heat flow equation?

7. State change of scale property on Fourier transforms.

8. Find the infinite Fourier sine transform of $f(x) = \frac{1}{x}$.

9. Find the Z - transform of a^n.

10. State initial and final value theorem on Z -transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the partial differential equations of all planes which are at a constant distance 'k' units from the origin. (8)

(ii) Solve the Lagrange's equation $x(z^2 - y^2)\, p + y(x^2 - z^2)\, q = z(y^2 - x^2)$. (8)

Or

(b) (i) Form the PDE by eliminating the arbitrary functions 'f' and 'φ' from the relation $z = xf\left(\frac{y}{x}\right) + y\varphi\,(x)$. (8)

(ii) Solve $(D^2 + DD' - 6D'^2)\, z = y \cos x$. (8)

12. (a) (i) Find the half range sine series of $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$.

Hence deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (10)

(ii) Find the complex form of the Fourier series o f$f(x) = e^{-x}$ in $-1 < x < 1$. (6)

Or

(b) (i) Find the Fourier series of $f(x) = |\sin x|$ in $-\pi < x < \pi$ of periodicity 2π. (8)

(ii) Compute upto the first three harmonics of the Fourier series of $f(x)$ given by the following table : (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

13. (a) A trightly stretched string of length $2l$ is fastened at $x = 0$ and $x = 2l$. The midpoint of the string is then pulled to height 'b' transversely and then released from rest in that positions. Find the lateral displacement of the string. (16)

Or

(b) A rectangular plate with insulated surface is 20 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature while the other short edge $x = 0$ is given by $u = \begin{cases} 10\ y\ \ for\ 0 \le y \le 10 \\ 10(20 - y)\ \ for\ 10 \le y \le 20 \end{cases}$ and the two long edges as well as the other short edge are kept at 0°C, find the steady state temperature distribution $u(x, y)$ in the plate. (16)

14. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0 & , |x| > 1 \end{cases}$ and hence deduce that $\int_0^\infty \left[\frac{\sin t}{t}\right]^4 dt = \frac{\pi}{3}$. (8)

(ii) Find the infinite Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ hence deduce the infinite Fourier sine transform of $\frac{1}{x}$. (8)

Or

(b) (i) Find the infinite Fourier transform of $e^{-a^2x^2}$ hence deduce the infinite Fourier transform of $e^{-x^2/2}$. (8)

(ii) Solve the integral equation $\int_0^\infty f(x) \cos \lambda x\ dx = e^{-\lambda}$, where $\lambda > 0$. (8)

15. (a) (i) Find :

(1) $Z[n^3]$

(2) $Z[e^{-t}t^2]$. (4+4)

(ii) Evaluate $Z^{-1}\left[\frac{9z^3}{(3z-1)^2\ (z-2)}\right]$, using calculus of residues. (8)

Or

(b) (i) Using convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$. (8)

(ii) Using Z transform, solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that $y_0 = y_1 = 0$. (8)
