

5. Write all possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.
6. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.
7. If $F(s)$ is the Fourier transform of $f(x)$, prove that $F\{f(x-a)\} = e^{ias}F(s)$.
8. Find Fourier Sine transform of $\frac{1}{x}$.
9. Find the Z-transform of a^n .
10. State initial and final value theorems on Z-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the general solution of $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. (8)
- (ii) Find the general solution of $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$. (8)

Or

- (b) (i) Find the general solution of $z = px + qy + p^2 + pq + q^2$. (8)
- (ii) Find the general solution of $(D^2 - 3DD' + 2D'^2 + 2D - 2D')z = \sin(2x + y)$. (8)
12. (a) (i) Find the Fourier series of period 2π for the function $f(x) = x \cos x$ in $0 < x < 2\pi$. (8)
- (ii) Find the Fourier series expansion for $y = f(x)$ up to second harmonic from the following data: (8)

$x:$	0	1	2	3	4	5
$y:$	9	18	24	28	26	20

Or

- (b) (i) Find the Fourier half-range cosine series of $f(x) = \begin{cases} x, & \text{in } 0 < x < 1 \\ 2-x, & \text{in } 1 < x < 2 \end{cases}$. (8)
- (ii) Find the complex form of the Fourier series of $f(x) = e^{-ax}$ in, $-l < x < l$. (8)

13. (a) A tightly stretched string of length $2l$ is fastened at $x=0$ and $x=2l$. The midpoint of the string is then taken to height ' b ' transversely and then released from rest in that position. Find the lateral displacement of the string. (16)

Or

- (b) A rectangular plate with insulated surfaces is 20 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature while the other short edge $x=0$ is given by $u = \begin{cases} 10y & \text{for } 0 \leq y \leq 10 \\ 10(20-y) & \text{for } 10 \leq y \leq 20 \end{cases}$ and the two long edges as well as the other short edge are kept at 0°C , find the steady state temperature distribution $u(x, y)$ in the plate. (16)

14. (a) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 & \text{for } |x| < 2 \\ 0 & \text{for } |x| > 2 \end{cases}$ and

hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$ and $\int_0^{\infty} \left(\frac{\sin x}{x}\right)^2 dx$. (16)

Or

- (b) (i) Find the Fourier cosine transform of $e^{-a^2x^2}$ for any $a > 0$. (8)

(ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$ using Fourier transforms. (8)

15. (a) (i) Find Z-transform of $\frac{2n+3}{(n+1)(n+2)}$. (8)

(ii) Using Convolution theorem, find $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$. (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{4z^3}{(2z-1)^2(z-1)}\right]$, by the method of partial fractions. (8)

(ii) Using Z-transforms, solve the equation $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$, given that $y_0 = y_1 = 0$. (8)