Reg. No. | | | | | | | | | | |

Question Paper Code : 57502

B.E./B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Third Semester

Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulations 2013)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Form the partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2, z - xy) = 0$.

2. Find the complete solution of the partial differential equation $p^3 - q^3 = 0$.

3. Find the value of the Fourier series of $f(x) = \begin{cases} 0 \text{ in } (-c, 0) \\ 1 \text{ in } (0, c) \end{cases}$ at the point of discontinuity $x = 0$.

4. Find the value of b_n in the Fourier series expansion of $f(x) = \begin{cases} x + \pi \text{ in } (-\pi, 0) \\ -x + \pi \text{ in } (0, \pi) \end{cases}$

5. Classify the partial differential equation $u_{xx} + u_{xy} = f(x, y)$.

6. Write down all the possible solutions of one dimensional heat equation.

7. State Fourier integral theorem.

8. Find the Fourier transform of a derivative of the function $f(x)$ if $f(x) \to 0$ as $x \to \pm \infty$.

9. Find $Z\left\{\frac{1}{n!}\right\}$

10. Find $Z\{(\cos\theta + i\sin\theta)^n\}$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Solve the equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. **(8)**

(ii) Find the singular integral of the equation $z = px + qy + \sqrt{1 + p^2 + q^2}$. **(8)**

OR

(b) (i) Solve : $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$. **(8)**

(ii) Solve : $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$ **(8)**

12. (a) (i) Find the Fourier series of $f(x) = x$ in $-\pi < x < \pi$. **(6)**

(ii) Find the Fourier series expansion of $f(x) = |\cos x|$ in $-\pi < x < \pi$. **(10)**

OR

(b) (i) Find the half range sine series of $f(x) = x\cos\pi x$ in $(0, 1)$. **(8)**

(ii) Find the Fourier cosine series up to third harmonic to represent the function given by the following data : **(8)**

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

13. (a) Find the displacement of a string stretched between two fixed points at a distance of $2l$ apart when the string is initially at rest in equilibrium position and points of the string are given initial velocities v where $v = \begin{cases} \frac{x}{l} & \text{in } (0, l) \\ \frac{2l - x}{l} & \text{in } (l, 2l) \end{cases}$, x being the distance measured from one end. **(16)**

OR

(b) A long rectangular plate with insulated surface is l cm wide. If the temperature along one short edge is $u(x, 0) = k(lx - x^2)$ for $0 < x < l$, while the other two long edges $x = 0$ and $x = 1$ as well as the other short edge are kept at 0 °C, find the steady state temperature function $u(x, y)$. **(16)**

14. (a) Find the Fourier cosine and sine transform of $f(x) = e^{-ax}$ for $x \geq 0$, $a > 0$. Hence deduce the integrals $\int_0^\infty \frac{\cos sx}{a^2 + s^2} ds$ and $\int_0^\infty \frac{s \sin sx}{a^2 + s^2} ds$. **(16)**

OR

(b) (i) Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$ in $(-\infty, \infty)$. **(8)**

(ii) Find the Fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$ and hence find the value of $\int_0^\infty \frac{\sin^4 t}{t^4} dt$. **(8)**

15. (a) (i) Find the Z-transforms of $\cos \frac{n\pi}{2}$ and $\frac{1}{n(n+1)}$. (8)

(ii) Using convolution theorem, evaluation $Z^{-1}\left\{\frac{z^2}{(z-a)^2}\right\}$. (8)

OR

(b) (i) Find the inverse Z-transform of $\frac{z}{z^2 - 2z + 2}$ by residue method. (8)

(ii) Solve the difference equation $y_{n+2} + y_n = 2$, given that $y_0 = 0$ and $y_1 = 0$ by using Z-transforms. (8)