

Reg. No.

Question Paper Code: 57502

B.E./B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Third Semester

Civil Engineering

MA 6351 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulations 2013)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions.

 $PART - A (10 \times 2 = 20 Marks)$

- 1. Form the partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2, z xy) = 0.$
- 2. Find the complete solution of the partial differential equation $p^3 q^3 = 0$.
- 3. Find the value of the Fourier series of $f(x) = \begin{cases} 0 & \text{in } (-c, 0) \\ 1 & \text{in } (0, c) \end{cases}$ at the point of discontinuity x = 0.
- 4. Find the value of b_n in the Fourier series expansion of $f(x) = \begin{cases} x + \pi & \text{in } (-\pi, 0) \\ -x + \pi & \text{in } (0, \pi) \end{cases}$

- 5. Classify the partial differential equation $u_{xx} + u_{xy} = f(x, y)$.
- 6. Write down all the possible solutions of one dimensional heat equation.
- 7. State Fourier integral theorem.
- 8. Find the Fourier transform of a derivative of the function f(x) if $f(x) \to 0$ as $x \to \pm \infty$.
- 9. Find $Z\left\{\frac{1}{n!}\right\}$
- 10. Find $Z \{(\cos \theta + i \sin \theta)^n\}$.

$PART - B (5 \times 16 = 80 Marks)$

11. (a) (i) Solve the equation
$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$
. (8)

(ii) Find the singular integral of the equation
$$z = px + qy + \sqrt{1 + p^2 + q^2}$$
. (8)

OR

(b) (i) Solve:
$$(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$$
. (8)

(ii) Solve:
$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$$
 (8)

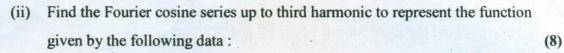
12. (a) (i) Find the Fourier series of
$$f(x) = x$$
 in $-\pi < x < \pi$. (6)

(ii) Find the Fourier series expansion of
$$f(x) = |\cos x| \text{ in } -\pi < x < \pi$$
. (10)

OR

(b) (i) Find the half range sine series of $f(x) = x \cos \pi x$ in (0, 1).

2 57502



13. (a) Find the displacement of a string stretched between two fixed points at a distance of 2*l* apart when the string is initially at rest in equilibrium position and points of

the string are given initial velocities
$$v$$
 where $v = \begin{cases} \frac{x}{l} & \text{in } (0, l) \\ \frac{2l - x}{l} & \text{in } (l, 2l) \end{cases}$, x being the

distance measured from one end.

(16)

OR

- (b) A long rectangular plate with insulated surface is l cm wide. If the temperature along one short edge is u(x, 0) = k(lx x²) for 0 < x < l, while the other two long edges x = 0 and x = 1 as well as the other short edge are kept at 0 °C, find the steady state temperature function u(x, y).
- 14. (a) Find the Fourier cosine and sine transform of $f(x) = e^{-ax}$ for $x \ge 0$, a > 0. Hence

deduce the integrals
$$\int_{0}^{\infty} \frac{\cos sx}{a^2 + s^2} ds$$
 and $\int_{0}^{\infty} \frac{s \sin sx}{a^2 + s^2} ds$. (16)

OR

(b) (i) Find the Fourier transform of
$$f(x) = e^{-\frac{x^2}{2}}$$
 in $(-\infty, \infty)$. (8)

(ii) Find the Fourier transform of f(x) = 1 - |x| if |x| < 1 and hence find the

value of
$$\int_{0}^{\infty} \frac{\sin^4 t}{t^4} dt$$
. (8)

3

- 15. (a) (i) Find the Z-transforms of $\cos \frac{n\pi}{2}$ and $\frac{1}{n(n+1)}$. (8)
 - (ii) Using convolution theorem, evaluation $Z^{-1}\left\{\frac{z^2}{(z-a)^2}\right\}$. (8)

OR

- (b) (i) Find the inverse Z-transform of $\frac{z}{z^2 2z + 2}$ by residue method. (8)
 - (ii) Solve the difference equation $y_{n+2} + y_n = 2$, given that $y_0 = 0$ and $y_1 = 0$ by using Z-transforms. (8)