ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2010

REGULATIONS : 2008

THIRD SEMESTER

080100008 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(COMMON TO CIVIL / EEE / EIE / ICE / ECE / BIOMEDICAL / BIOTECH / AERO / AUTO / CSE / IT / MECHANICAL / CHEMICAL / FT / TT / TC)

Time: 3 Hours **Max.Marks: 100**

PART - A

(20 x 2 = 40 Marks)

ANSWER ALL QUESTIONS

1. State the conditions for f(x) to have Fourier series expansion.
2. Write a_0, a_n in the expansion of $x+x^3$ as Fourier Series in $(-\pi, \pi)$.
3. Expand f(x)=1 in a sine series in $0<x<\pi$
4. Find Root Mean Square value of the function $f(x)=x$ in the interval $(0,l)$.
5. Define Fourier Transform Pair.
6. Find Fourier Cosine transform of e^{-2x}.
7. If F(S) is the Fourier Transform of $f(x)$, show that the Fourier Transform of $e^{iax} f(x)$ is $F(S+a)$.
8. State Parseval's Identity for Fourier Transform.
9. Eliminate the arbitrary constants a & b from $z=(x^2+a)(y^2+b)$.
10. Form the PDE by eliminating the functions from $z=f(x+t)+g(x-t)$.
11. Find the complete integral $q=2px$.
12. Solve $(D^3-3DD'^2+2D'^3)z=0$.
13. Find the nature of PDE $4u_{xx}+4u_{xy}+u_{yy}+2u_x-u_y=0$.
14. What are the various solutions of one dimensional Wave Equation?
15. A string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form $y=y_0 \sin(\frac{\pi x}{l})$from which it is released at time t=0. Formulate this problem as the boundary value problem.
16. A rod of length 20cm whose one end is kept at 30^0C and the other end is kept at 70^0C is maintained so until steady state prevails. Find the steady state temperature.
17. Find $Z[e^{-an}]$.
18. Prove that $Z[n]=\frac{z}{(z-1)^2}$
19. Prove that $Z[f(n+1)]=zF(z)-zf(0)$
20. State Initial and Final value theorem on Z- transform.

PART – B

(5 x 12 = 60 Marks)

ANSWER ANY FIVE QUESTIONS

21(a). If $f(x)=\left(\frac{\pi-x}{2}\right)$ find the Fourier Series of the period 2π in the interval $(0,2\pi)$.

Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....=\frac{\pi}{4}$ (8)

(b). Find the Fourier expansion of $f(x)=x$ in the interval $(-\pi,\pi)$ (4)

22. Show that the Fourier Transform of $f(x)=\begin{cases} a^2-x^2 & |x|\le a \\ 0 & otherwise \end{cases}$ is

$2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - as\cos as}{s^3}\right)$ Hence deduce that $\int_0^{\infty}\frac{\sin t - t\cos t}{t^3}dt=\frac{\pi}{4}$

Using Parseval's Identity show that $\int_0^{\infty}\left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt=\frac{\pi}{15}$

23.(a) Solve (mz-ny)p + (nx-lz)q = ly-mx (6)

(b) Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2Y$ (6)

24. A string of length l is initially at rest in its equillibrium position and motion is started by giving each of its points is given a velocity $V=\begin{cases} cx & , 0\le x\le l/2 \\ c(l-x), & l/2\le x\le l \end{cases}$

Find the displacement function y(x,t).

25 (a) Evaluate $z^{-1}\left[\frac{z}{z^2+7z+10}\right]$ (6)

(b) Using z-transforms solve u(n+2) - 5u(n+1) + 6u(n)=4^n given that u(0)=0, u(1)=1 (6)

26(a) Find the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of, y=f(x) as given in the following table:- (6)

x	0	1	2	3	4	5	6
f(x)	9	18	24	28	26	20	9

26(b) Find the Fourier transform of $f(x)=\begin{cases} 1-|x| & |x|\le 1 \\ 0 & otherwise \end{cases}$

hence find the value of $\int_0^{\infty}\frac{\sin^4 x}{x^4}dx$ (6)

27. A metal bar 30cm long has its ends A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature u(x,t) taking x=0 at A.

28(a) Solve $p(1+q)=qz$ (6)

(b) Using Convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ (6)

*****THE END*****