ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2011

REGULATIONS : 2008

THIRD SEMESTER

080100008 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATION

(COMMON TO AERO / AUTO / BIOMEDICAL / CIVIL / CSE / IT / EEE / EIE / ICE / ECE / MECHANICAL / BIOTECH. / CHEMICAL / FASHION TECH. / TEXTILE TECH. / TEXTILE CHEMISTRY / PRODUCTION ENGG.)

Time : 3 Hours Max.Marks : 100

PART - A

(10 x 2 = 20 Marks)

ANSWER ALL QUESTIONS

1. State Dirichlet's conditions.
2. If f(x) = sinx in $(-\pi, \pi)$, then find the values of a_0 and a_n
3. State Fourier integral theorem
4. Find the Fourier cosine transform of e^{-x}.
5. Find the singular integral of $z = px+qy+p^2$.
6. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = ax^3 + by^3$
7. Classify the equation $\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$
8. Write all the three possible solutions of one dimensional heat equation.
9. Find $Z\left[\frac{1}{n}\right]$
10. Prove that $Z[f(n+1)] = zF[z] - zf(0)$

PART - B

(5 x 16 = 80 Marks)

ANSWER ALL QUESTIONS

11. a) (i)Find the Fourier series for $f(x) = x^2$ in $(-\pi, \pi)$, hence deduce the value of $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots$ 8

(ii)Find the Fourier series upto second harmonics for the function y=f(x) in $(0, 2\pi)$ 8

x:	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$:	1	1.4	1.9	1.7	1.5	1.2	1

OR

b) (i)Find the Fourier sine series for the function $f(x) = x(\pi - x)$ in $(0, \pi)$, and hence deduce the value of $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \ldots$ (8)

(ii)Find the Fourier expansion of f(x) = x in the interval $(-\pi, \pi)$. (8)

12. a) Find the Fourier transform of $f(x)=\begin{cases}1-|x| & ,\text{ if } |x|<1\\ 0 & ,\text{ if } |x|>1\end{cases}$

Hence find the value of $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$

OR

12. b) (i)Find the Fourier cosine transform of $f(x)=\begin{cases}x & ,0<x<1\\ 2-x & ,1<x<2\\ 0 & ,x>2\end{cases}$ 8

(ii)Find the Fourier transform of $e^{\frac{-x^2}{2}}$ is $e^{\frac{-s^2}{2}}$ 8

13. a) (i)Solve $(D^2-DD'-30D'^2)z=xy+e^{6x+y}$ 8

(ii)Solve (3z – 4y)P + (4x – 2z) q = 2y -3x 8

OR

b) (i)Form the partial differential equation by eliminating the arbitrary function f and g from $z=f(x+ct)+g(x-ct)$ 8

(ii)Solve $z=1+p^2+q^2$ 8

14. a) A tightly stretched string with fixed end points x = 0 and x = l initially in a position given by $y(x,0)=y_0\sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this position, find the displacement y at any time and at any distance from the end x = 0.

OR

b) A square plate is bounded by the lines x = 0, y = 0, x = 20 and y = 20. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x,20) = x(20 - x) when 0 < x < 20, while the other three edges are kept at $0^\circ C$. Find the steady state temperature in the plate.

15. a) (i)Find $Z\left[\frac{1}{(n+1)(n+2)}\right]$ 8

(ii)Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ by method of partial fraction 8

OR

b) (i)Using Z-transform, solve 8

$y(n+2)-4y(n+1)+4y(n)=0$ where

$y(0)=1$ and $y(1)=0$

(ii)Using Convolution theorem evaluate $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ 8

*****THE END*****