Reg. No.

Question Paper Code : 51772

B.E/B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fifth/Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/080100008/080210001/10177 MA 301 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS / MATHEMATICS – III

(Common to all branches)

(Regulations 2008/2010)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Form the partial differential equations of all planes passing through the origin.

2. Find the complete integral of $\sqrt{p} + \sqrt{q} = 1$.

3. If $x^2 = \frac{\pi}{3} - 4 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos(nx)}{n^2}$ in $-\pi < x < \pi$, then find $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

4. State TRUE or FALSE : Fourier series of period 20 for the function $f(x) = x \cos(x)$ in the interval (–10, 10) contains only sine terms. Justify your answer.

5. Write down the initial and boundary conditions for the boundary value problem when a string of length l is tightly fastened on both ends and the midpoint of the string is taken to height of k are released from rest.

6. The ends A and B of a rod of length 20 cm have their temperature kept at 10 °C and 50 °C respectively. Find the steady state temperature distribution on the rod.

7. If F(s) is the Fourier transform of $f(x)$, obtain the Fourier transform of $f(x-2) + f(x+2)$.

8. Given that $F_S\{f(x)\} = \dfrac{s}{s^2 + a^2}$ for $a > 0$, hence find $F_C\{xf(x)\}$.

9. If $Z\{n^2\} = \dfrac{z^2 + z}{(z-1)^3}$, then find $Z\{(n+1)^2\}$

10. State damping rule related to Z-transform and then find $Z(na^n)$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Find the general solution of the equation :

$$x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2).$$ **(8)**

(ii) Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y) + 3e^{2x+y}$. **(8)**

OR

(b) (i) Form the partial differential equations by eliminating the arbitrary function

$f(x + y + z, x^2 + y^2 + z^2) = 0$ **(8)**

(ii) Obtain the complete integral of $p^2 + x^2y^2q^2 = x^2z^2$. **(8)**

12. (a) (i) Find the Fourier series for

$$f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce the sum of the series $1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots$ to ∞. **(8)**

(ii) Find the Fourier series up to second harmonic to represent the function given by the following discrete data : **(8)**

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

OR

(b) (i) Find the half-range Fourier Cosine series expansion for the function $f(x) = x$ in $0 < x < l$. Hence deduce the sum of the series $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$. **(8)**

(ii) Find the complex form of the Fourier series of $f(x) = e^{-x}, -1 < x < 1$. **(8)**

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially displaced to the form $2 \sin\left(\frac{3\pi x}{l}\right) \cos\left(\frac{2\pi x}{l}\right)$ and then released. Find the displacement of the string at any distance x from one end at any time t. **(16)**

OR

(b) A rectangular plate is bounded by the lines $x = 0, x = a, y = 0$ and $y = b$ and the edge temperatures are $u(x, 0) = 10 \sin\left(\frac{3\pi x}{a}\right) + 8 \sin\left(\frac{5\pi x}{a}\right)$, $u(0, y) = 0$, $u(x, b) = 0$ and $u(a, y) = 0$. Find the steady state temperature distribution $u(x, y)$ at any point of the plate. **(16)**

14. (a) (i) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \left(\frac{\sin s - s \cos s}{s^3}\right) \cos\left(\frac{s}{2}\right) ds$. **(8)**

(ii) Find the Fourier Sine transform of $f(x) = e^{-ax}$ and hence find Fourier Sine transform $\frac{x}{a^2 + x^2}$ **(8)**

OR

(b) (i) Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a > 0 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$ **(8)**

(ii) Find the Fourier Cosine transform of $f(x) = e^{-ax}$. Hence, evaluate the following :

$$\int_0^\infty \frac{1}{(x^2 + a^2)(x^2 + b^2)} dx$$ **(8)**

15. (a) (i) Derive a difference equation by eliminating the constants from $y_n = (A + Bn)3^n$. **(8)**

(ii) Use convolution theorem to find the inverse Z-transform of $\frac{z^2}{(z - 1/2)(z - 1/4)}$ **(8)**

OR

(b) (i) State initial value theorem. Use it to find u_0, u_1, u_2 and u_3, where

$$U(z) = \frac{2z^2 + 5z + 14}{(z - 1)^4}$$ **(8)**

(ii) Solve the equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = y_1 = 0$. **(8)**