Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 60772

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Third/Fifth Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/080100008/080210001/10177 MA 301 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS/ MATHEMATICS — III

(Common to all branches)

(Regulations 2008/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If $f(x) = x^2 + x$ is expressed as a Fourier series in the interval $-2 < x < 2$, to which value this series converges at $x = 2$.

2. Define R.M.S. value of a function $f(x)$ in $c < x < c + 2l$.

3. Define Fourier Transform pair.

4. Find the Fourier cosine transform of e^{-ax}, $a > 0$.

5. Find the partial differential equation by eliminating f from $z = f(x^2 + y^2)$.

6. Solve $p \tan x + q \tan y = \tan z$.

7. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial^2 u}{\partial x \partial y}$, $x > 0$, $y > 0$.

8. Write down all possible solutions of one dimensional heat flow equation.

9. Find the Z-transform of n^2.

10. State the initial and final value theorems of Z-transforms.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Expand $f(x) = e^{ax}$ as a Fourier series in $(0, 2\pi)$. (10)

(ii) Find the half-range sine series expansion for $f(x) = k(lx - x^2)$ in $(0, l)$. (6)

Or

(b) (i) Find the Fourier series expansion for $f(x) = \begin{cases} l + x, & -l \le x \le 0 \\ l - x, & 0 \le x \le l \end{cases}$ and hence prove that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. (8)

(ii) Determine the first two harmonic of the Fourier series for the following data : (8)

x :	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	1.0

12. (a) (i) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 - x^2, \text{for } |x| < 1 \\ 0, \quad \text{for } |x| > 1 \end{cases}$. Hence evaluate $\int_0^{\infty} \left(\frac{\sin x - x \cos x}{x^3} \right) \cos \frac{x}{2} dx$. (10)

(ii) Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$. (6)

Or

(b) (i) Find the Fourier transform of $xe^{-a|x|}$, $a > 0$. (8)

(ii) Prove that the Fourier sine transform of a sine transform of a given function is itself. Hence find the Fourier sine transform of $\frac{x}{x^2 + a^2}$. (8)

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t. (16)

Or

(b) A square plate is bounded by the lines $x = 0$, $y = 0$, $x = l$ and $y = l$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, l) = lx - x^2$, $0 < x < l$ while the other three edges are kept at 0°C. Find the steady state temperature in the plate. (16)

14. (a) (i) Find the differential equation of all spheres whose radii are the same. (6)

(ii) Solve : $(D^3 - 7DD'^2 - 6D'^3)z = e^{3x+y} + \sin(x + 2y)$. (10)

Or

(b) (i) Find the singular integral of $z = px + qy + p^2 + q^2$. (8)

(ii) Solve : $(mz - ny)p + (nx - lz)q = ly - mx$. (8)

15. (a) (i) Find the Z-transform of $\frac{2n+3}{(n+1)(n+2)}$. (6)

(ii) Solve the difference equation $y(n+2) - 3y(n+1) + 2y(n) = 2^n$, given that $y(0) = 3$ and $y(1) = 6$, using Z-transform method. (10)

Or

(b) (i) Find $Z^{-1}\left\{\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}\right\}$ by the partial fraction method. (8)

(ii) Find the Z-transform of $\sin^2\left(\frac{n\pi}{4}\right)$ and $\cos^2 t$. (8)
