15. (a) (i) Solve $(D^2+4D+5)y = e^x + x^3 + \cos 2x + 1$. (8)

(ii) Solve $\frac{dx}{dt} - \frac{dy}{dt} + 2y = \cos 2t$, $\frac{dx}{dt} - 2x + \frac{dy}{dt} = \sin 2t$. (8)

Or

(b) (i) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \left(\frac{\ln x}{x}\right)^2$. (8)

(ii) Solve $y'' - 4y' + 4y = (x+1)e^{2x}$ by the method of variation of parameters. (8)

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Reg. No. : [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

Question Paper Code : 25201

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

First Semester

Civil Engineering

MA 8151 — MATHEMATICS — I

(Common to Mechanical Engineering (Sandwich), Aeronautical Engineering, Agriculture Engineering, Automobile Engineering, Biomedical Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering, Environmental Engineering, Geoinformatics Engineering, Industrial Engineering, Industrial Engineering and Management, Instrumentation and Control Engineering, Manufacturing Engineering, Materials Science and Engineering, Mechanical Engineering, Mechanical and Automation Engineering, Mechatronics Engineering, Medical Electronics Engineering, Petrochemical Engineering, Production Engineering, Robotics and Automation Engineering, Biotechnology, Chemical Engineering, Chemical and Electrochemical Engineering, Fashion Technology, Food Technology, Handloom and Textile Technology, Information Technology, Petrochemical Technology, Petroleum Engineering, Pharmaceutical Technology, Plastic Technology, Polymer Technology, Textile Chemistry, Textile Technology, Textile Technology (Fashion Technology)
Except Marine Engineering)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the domain of $f(x) = \sqrt{3-x} - \sqrt{2+x}$.

2. Evaluate $\lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1}$.

3. Find $\frac{dy}{dx}$, if $x^y + y^x = c$, where c is a constant.

4. State the properties of Jacobians.

5. State the fundamental theorem of calculus.

6. If f is continuous and $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$.

7. Evaluate $\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dx\,dy$.

8. Change the order of integration in $\int_0^1\int_{y^2}^{y} f(x,y)\,dx\,dy$.

9. Solve $(D^3+1)y=0$.

10. Transform the equation $xy''+y'+1=0$ into a linear equation with constant coefficients.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Guess the value of the limit (if it exists) for the function $\lim_{x\to 0}\frac{e^{5x}-1}{x}$ by evaluating the function at the given numbers $x=\pm 0.5$, ± 0.1, ± 0.01, ± 0.001, ± 0.0001 (correct to six decimal places) (6)

(ii) For the function $f(x)=2+2x^2-x^4$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points. (10)

Or

(b) (i) Find the values of a and b that make f continuous on $(-\infty,\infty)$.

$$f(x)=\begin{cases}\frac{x^3-8}{x-2}, & if\ x<2\\ ax^2-bx+3, & if\ 2\le x<3\\ 2x-a+b, & if\ x\ge 3\end{cases}$$ (8)

(ii) Find the derivative of $f(x)=\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$. (4)

(iii) Find y' for $\cos(xy)=1+\sin y$. (4)

12. (a) (i) If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$, find $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}$. (8)

(ii) Find the maxima and minima of $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$. (8)

Or

(b) (i) Find Taylor's series expansion of function of $f(x)=\sqrt{1+x+y^2}$ in powers of $(x-1)$ and y up to second degree terms. (8)

(ii) Find the minimum distance from the point (1, 2, 0) to the cone $z^2=x^2+y^2$. (8)

13. (a) (i) Using integration by parts, evaluate $\int\frac{(\ln x)^2}{x^2}dx$. (8)

(ii) Evaluate $\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}}\frac{dx}{x^5\sqrt{9x^2-1}}$. (8)

Or

(b) (i) Establish a reduction formula for $I_n=\int\sin^n x\,dx$. Hence, find $\int_0^{\frac{\pi}{2}}\sin^n x\,dx$. (10)

(ii) For what values of p is $\int_1^{\infty}\frac{1}{x^p}dx$ convergent? (6)

14. (a) (i) Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y=x^2$ and $y=x$. (8)

(ii) Express $\int_0^a\int_y^a\frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}dx\,dy$ in polar coordinates and then evaluate it. (8)

Or

(b) (i) Find the area bounded by the parabolas $y^2=4-x$ and $y^2=x$. (8)

(ii) Evaluate $\iiint_V dx\,dy\,dz$, where V is the finite region of space (tetrahedron) bounded by the planes $x=0$, $y=0$, $z=0$ and $2x+3y+4z=12$. (8)