Reg. No. :

Question Paper Code : 40057

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Second Semester

Aeronautical Engineering

MA 8251 – ENGINEERING MATHEMATICS – II

(Common to all branches, except Marine Engineering)

(Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. If 3 and 5 are two eigenvalues of the matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ then find its third eigenvalue and hence $|A|$.

2. Show that the eigenvalues of a null matrix are zero.

3. If $\vec{F} = x^3\,\vec{i} + y^3\vec{j} + z^3\,\vec{k}$, then find div curl $\vec{F}$.

4. Find the values of a, b, c such that the following vector is irrotational.
$\vec{F} = (x + 2y + az)\,\vec{i} + (bx - 3y - z)\,\vec{j} + (4x + cy + 2z)\vec{k}$.

5. If $f(z) = r^2 (\cos 2\theta + i \sin p\theta)$ is analytic, then find the value of 'p'.

6. Examine whether the function $u = xy^2$ can be a real part of an analytic function.

7. If 'C' is the circle $|z| = 3$ and if $g(z_0) = \int_C \frac{2z^2 - z - 2}{z - z_0} dz$ then find g(2).

8. Find the value of $\int_C \frac{3z^2 + 7z + 1}{z + 1} dz$ if C is $|z| = \frac{1}{2}$.

9. If $L[f(t)] = F(s)$ then prove that $L[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$.

10. Find the Laplace transform of $\left[\frac{t}{e^t}\right]$.

PART – B (5×16 = 80 Marks)

11. a) i) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ (8)

ii) Using Cayley-Hamilton theorem find the inverse of the given matrix

$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$. (8)

(OR)

b) Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to a canonical form through an orthogonal transformation. Find also its nature. (16)

12. a) Verify the Gauss divergence theorem for $\vec{F} = x^3\,\vec{i} + y^3\,\vec{j} + z^3\,\vec{k}$ taken over the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0$ and $z = a$. (16)

(OR)

b) Verify Stoke's theorem for $\vec{F} = (y - z + 2)\,\vec{i} + (yz + 4)\,\vec{j} - (xz)\,\vec{k}$ where S is the open surface of the cube $x = 0, x = 2, y = 0, y = 2, z = 0$ and $z = 2$ above the xy -plane. (16)

13. a) i) Find the analytic function $f(z) = u + iv$ if $u - v = e^x[\cos y - \sin y]$. (8)

ii) Find the bilinear transformation which maps the points $z = -1, 0, 1$ on to the points $w = -1, -i, 1$. Show that under this transformation the upper half of the z-plane maps on to the interior of the unit circle $|w| = 1$. (8)

(OR)

b) i) If $f(z) = u + iv$ is an analytic function then prove that

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(u^p) = p(p-1)(u^{p-2})\,|f'(z)|^2$. (8)

ii) Find the image of the circle $|z - 2i| = 2$ in the complex plane under the transformation $w = \frac{1}{z}$. (8)

14. a) i) Evaluate $\int_C \frac{z^2}{(z^2+1)^2}\,dz$ where C is the circle $|z - i| = 1$ by using Cauchy's integral formula. (8)

ii) Expand $f(z) = \frac{6z+5}{(z+1)\,z\,(z-2)}$ in Laurent's series valid for $1 < |z+1| < 3$. (8)

(OR)

b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\,d\theta$ using contour integration. (16)

15. a) i) Using convolution theorem find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$. (8)

ii) Find the Laplace transform of $[t \cos t \sinh 2t]$. (8)

(OR)

b) i) Find $L[f(t)]$ if $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ given $f(t+2) = f(t)$. (8)

ii) Solve $y'' - 3y' + 2y = 1$ given that $y(0) = 0, y'(0) = 1$ by using Laplace transform method. (8)