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Reg. No. : [ ]

Question Paper Code : 80214

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third Semester

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Agriculture Engineering/Automobile Engineering/Civil Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and control Engineering/Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering(Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Production Engineering/Robotics and Automation Engineering/Biotechnology/Chemical and Electrochemical Engineering/ Food Technology/Pharmaceutical Technology)

(Regulation 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form the partial differential equation from the equation $2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$.

2. If $u = x^2 + t^2$ is a solution of $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, then find the value of c ?

3. State giving reasons whether the function $f(x) = x \sin\left(\frac{1}{x}\right)$ can be expanded in Fourier series in the interval of $(0, 2\pi)$.

4. Sketch the graph of one even and one odd extension of $f(x) = x^3$ in $[0, 1]$.

5. Classify the PDE $3\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

6. Write all three possible solutions of one dimensional heat equations.

7. State convolution theorem for Fourier transform.

8. State the condition for the existence of Fourier cosine and sine transforms of derivatives.

9. The integers 0, 1, 1, 2, 3, 5, 8, ... are said to form a Fibonacci sequence. Model the Fibonacci difference equation (no need to solve)

10. Find Z-transform of unit impulse sequence $\delta(n)=\begin{cases}1, & n=0\\ 0, & n\neq 0\end{cases}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2 \partial y}=2e^{2x}+3x^2y$.

(ii) Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$, where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$. (8 + 8)

Or

(b) (i) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x dy}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$.

(ii) Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$ where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$. (8 + 8)

12. (a) Find the Fourier series expansion of $f(x)=\sqrt{1-\cos x}$, $0\le x\le 2\pi$ and hence evaluate the value of the series $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}-\ldots$

Or

(b) The displacement $y(x)$ of a part of a mechanism is tabulated with corresponding angular movement $x°$ of the crank. Express $y(x)$ as a Fourier series neglecting the harmonics above the third.

$x°$:	0	30	60	90	120	150	180	210	240	270	300	330
$y(x)$:	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2

13. (a) (i) Using the method of separation of variables, solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ where $u(x,0)=6e^{-3x}$.

(ii) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x,0)=3\sin\pi x$, $u(0,t)=0$ and $u(1,t)=0$ where $0<x<1$, $t>0$. (8 + 8)

Or

(b) (i) Solve using by the method of separation of variables $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$.

(ii) A tightly stretched flexible string has its ends fixed at $x=0$ and $x=L$. At time $t=0$, the string is given a shape defined by $y=\mu x(L-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at any time $t>0$. (8 + 8)

14. (a) Find the Fourier transform of $e^{-a^2x^2}$, $a>0$. By using the properties, find the Fourier transform of $e^{-2(x-3)^2}$. (10 + 6)

Or

(b) Using Parseval's identities, prove that

(i) $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)}=\frac{\pi}{2ab(a+b)}$ (ii) $\int_0^\infty \frac{t^2 dt}{(t^2+1)^2}=\frac{\pi}{4}$. (8 + 8)

15. (a) Find the inverse Z-transform of

(i) $\frac{2z^2+3z}{(z+2)(z-4)}$ (ii) $\frac{2(z^2-5z+6.5)}{(z-2)(z-3)^2}$ for $2<|z|<3$. (6 + 10)

Or

(b) Using the Z-transform, solve

(i) $u_{n+2}+4u_{n+1}+3u_n=3^n$ with $u_0=0$, $u_1=1$

(ii) $u_{n+2}-2u_{n+1}+u_n=3n+5$. (8 + 8)