Question Paper Code : 90338

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Third Semester

Civil Engineering

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Aerospace Engineering/Agriculture Engineering/Automobile Engineering/Electrical and Electronics Engineering/ Electronics and Instrumentation Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/ Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/ Mechanical and Automation Engineering/Mechatronics Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical and Electrochemical Engineering/Food Technology/Pharmaceutical Technology)

(Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A
(10×2=20 Marks)

1. Find the complete solution of $p = 2qx$.

2. Solve $(D^2 - 6DD' + 9D'^2) z = 0$.

3. State the Dirichlet's conditions.

4. Sketch the even extension of the function $f(x) = \sin x$, $0 < x < \pi$.

5. Classify the two-dimensional steady state heat conduction equation.

6. Give the mathematical formulation of the problem of one-dimensional heat conduction in a rod of length l with insulated ends and with initial temperature $f(x)$.

7. State the convolution theorem for Fourier Transforms.

8. Show that $\Im_c\left[f(x)\cos ax\right] = \frac{1}{2}\{F_c(s+a)+F_c(s-a)\}$ where $\Im_c\left[f(x)\right] = F_c(s)$ is the Fourier cosine transform of f (x).

9. Show that $Z\left[a^n\ f(n)\right] = F\left(\frac{z}{a}\right)$ where $Z\left[f(n)\right] = F(z)$ is the Z-transform of f (x).

10. State the initial and final value theorems of Z-transforms.

<div align="center">

PART – B   (5×16=80 Marks)

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11. a) i) Solve $(D^3 - 2D^2\ D')\ z = \sin(x+2y) + 3x^2\ y$.   (10)

    ii) Form the partial differential equation by eliminating the arbitrary functions from u = f (x + ct) + g (x – ct).   (6)

<div align="center">(OR)</div>

b) i) Solve $(x^2 - yz)\ p + (y^2 - zx)\ q = (z^2 - xy)$.   (10)

    ii) Solve $p - x^2 = q + y^2$.   (6)

12. a) i) Find the Fourier series of f (x) = x^2 in (0, 2l). Hence deduce that
$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\infty = \frac{\pi^2}{6}.$$
  (10)

    ii) Find the complex form of the Fourier series of f (x) = cos ax in $(-\pi, \pi)$, where 'a' is neither zero nor an integer.   (6)

<div align="center">(OR)</div>

b) i) Obtain the constant term and the first three harmonics in the Fourier Cosine series of y = f (x) in (0, 6) from the following table.   (10)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

    ii) Find the Fourier series expansion of f (x) = sin ax in (–l, l).   (6)

13. a) i) Solve $u_t = a^2\ u_{xx}$ by the method of separation of variables and obtain all possible solutions.   (8)

    ii) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate.

    If the temperature along the short edge y = 0 is u (x, 0) = 100 sin $\left(\frac{\pi x}{8}\right)$, 0 < x < 8 while two long edges x = 0 & x = 8 as well as the other short edge are kept at 0°C, then find the steady state temperature at any point of the plate.   (8)

<div align="center">(OR)</div>

b) i) Solve the problem of a tightly stretched string with fixed end points x = 0 & x = 1 which is initially in the position y = f (x) and which is initially set vibrating by giving to each of its points a velocity $\frac{dy}{dt}$ = g (x) at t = 0.   (10)

    ii) Classify the partial differential equation
$(1 - x^2)\ f_{xx} - 2xyf_{xy} + (1 - y^2)\ f_{yy} = 0$.   (6)

14. a) i) Find the Fourier transform of f (x) where f (x) = $\begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x}\ dx$.   (10)

    ii) Show that $\frac{1}{\sqrt{x}}$ is self-reciprocal under the Fourier cosine transform.   (6)

<div align="center">(OR)</div>

b) i) Find the Fourier cosine and sine transforms of e^{-ax}, a > 0 and hence deduce their inversion formulae.   (10)

    ii) Using Parseval's identity, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$, a > 0.   (6)

15. a) i) Find Z {sin bt} and hence find Z {e^{-at} sin bt}.   (8)

    ii) Find $Z^{-1}\left\{\dfrac{8z^2}{(2z-1)(4z+1)}\right\}$ using convolution theorem.   (8)

<div align="center">(OR)</div>

b) i) Using Z-transforms, solve the difference equation $y_{n+2} - 7\ y_{n+1} + 12y_n = 2^n$ given $y_0 = y_1 = 0$. Use partial fraction method to find the inverse Z-transform.   (8)

    ii) Using residue method, find $Z^{-1}\left\{\dfrac{z}{z^2+2z+2}\right\}$.   (8)