15. (a) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ with $u = 0$ on the boundary and mesh length 1 unit.

Or

(b) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units ; satisfying the following boundary conditions.

(i) $u(0, y) = 0$ for $0 \le y \le 4$

(ii) $u(4, y) = 12 + y$ for $0 \le y \le 4$

(iii) $u(x, 0) = 3x$ for $0 \le x \le 4$

(iv) $u(x, 4) = x^2$ for $0 \le x \le 4$

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08/07/22 - FN.

Reg. No. :

Question Paper Code : 20818

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Fourth/Fifth/Sixth Semester

Aeronautical Engineering

MA 8491 — NUMERICAL METHODS

(Common to Aerospace Engineering/Agriculture Engineering/Civil Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Instrumentation and Control Engineering/Manufacturing Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Plastic Technology/Polymer Technology/Textile Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write a sufficient condition for Gauss-Seidel method to converge.

2. What is the order of convergence of Newton-Raphson method?

3. Find the first divided difference values for the following data.

X	–3	–1	0	3	5
Y	–30	–22	–12	330	3458

4. Is it possible to find two different interpolants to the same $(n+1)$ data using Lagrange's interpolation method? Justify.

5. How the accuracy can be increased in Trapezoidal rule of evaluating a given definite integral?

6. What is the error in Simpson's 1/3 rule in (x_0, x_2)?

7. Given $y' = x + y$, $y(1) = 0$ find $y(1.1)$ by Taylor's method.

8. What will you do, if there is a considerable difference between predicted value and corrected value in predictor corrector methods?

9. In the one dimensional heat equation $u_t = \alpha^2 u_{xx}$, what is α^2?

10. What is the condition for the partial differential equation $af_{xx} + bf_{xy} + cf_{yy} + df_x + ef_y + uf = k$ to represent a hyperbolic equation, elliptic and parabola?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the real root between 1 and 2 of the equation $2x^3 - 3x - 6 = 0$ by applying Newton-Raphson's method, correct to five decimal places. (8)

(ii) Using power method, determine the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$; Let initial vector be $X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. (8)

Or

(b) Solve the following system of equations by

(i) Gauss Jacobi method

(ii) Gauss Seidel method

$8x - 3y + 2z = 20;\ 4x + 11y - z = 33;\ 6x + 3y + 12z = 35$.

12. (a) (i) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface.

X height :	100	150	200	250	300	350	400
Y distance :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the value of y when $x = 218$ ft by using Newton's forward interpolation formula. (8)

(ii) Using Lagrange's formula of interpolation, find $y(9.5)$ given. (8)

x	7	8	9	10
y	3	1	1	9

Or

(b) (i) Form the difference table and using Newton's backward interpolation formula, compute $y(17)$ from the following data. (8)

X:	8	10	12	14	16	18
Y:	10	19	32.5	54	89.5	15.4

(ii) The following are the values of x and y :

X:	1	2	3	4
Y:	1	2	5	11

Find the cubic splines and evaluate $y(1.5)$. (8)

13. (a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by (i) Trapezoidal rule (ii) Simpson 1/3 rule (iii) Also check up the results by actual integration. Assume h = 1.

Or

(b) The population of a certain town is given below. Find the rate of growth of the population in (i) 1931 and (ii) 1971 by using Newton's forward and backward formulae respectively.

Year X :	1931	1941	1951	1961	1971
Population Y :	40.62	60.80	79.95	103.56	132.65

14. (a) Given $\frac{dy}{dx} = 1 - y;\ y(0) = 0$ and $y(0.3) = 0.2629$. Find

(i) $y(0.1)$ using Euler's method

(ii) $y(0.2)$ by Modified Euler's method

(iii) $y(0.4)$ by Milne's method.

Or

(b) Given $\frac{dy}{dx} = xy + y^2$; $y(0) = 1$. Find $y(0.1), y(0.2), y(0.3)$ by using Runge-Kutta method of order four and hence obtain $y(0.4)$ by using Adam's method.