(b) (i) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ by Euler's method to find $y(0.1)$ with $h = 0.05$. (6)

(ii) Solve $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$ and $y(0.6) = 0.1762$ by Milne's method to find $y(0.8)$. (10)

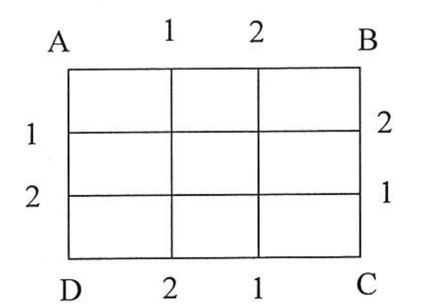
15. (a) (i) Solve the Poisson equation $u_{xx} + u_{yy} = -81\,xy$, for $0 < x,\ y < 1$, given $u(x, 0) = 0 = u(0, y)$, and $u(x, 1) = 100 = u(1, y)$. (8)

(ii) Use Crank-Nicholson implicit scheme to solve $16\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$ and $t > 0$ given $u(x, 0) = 0 = u(0, t)$, and $u(1,t) = 100t$. Compute $u(x, t)$ for one time step, taking $\Delta X = 0.25$. (8)

Or

(b) (i) Solve the boundary value problem $x\,y'' + y = 0$ with the boundary conditions $y(1) = 1$ and $y(2) = 2$, taking $h = 1/4$ by finite difference method. (8)

(ii) Solve $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in the figure below (8)



05/01/2023 - FN.

Reg. No. :

Question Paper Code : 90821

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Fourth/Fifth/Sixth Semester

Aeronautical Engineering

MA 8491 – NUMERICAL METHODS

(Common to : Aerospace Engineering/Agriculture Engineering/Civil Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Instrumentation and Control Engineering/Manufacturing Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Biotechnology and Biochemical Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Plastic Technology/Polymer Technology/Textile technology)

(Regulations 2017)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. State the sufficient condition for the convergence of Newton-Raphson method for the equation $f(x) = 0$.

2. State the principle used in Gauss Jordan method.

3. Find the second divided difference with arguments a, b and c of the function $f(x) = \frac{1}{x}$.

4. What are the advantages of Lagrange's formula over Newton's forward and backward interpolation formulae?

5. Write the formula for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_n$ by using Newton's backward difference operator.

6. What is the restriction on the number of intervals in order to evaluate $\int_a^b f(x)\,dx$ by Trapezoidal rule and by Simpson's one-third rule?

7. State the modified Euler formula to find $y(x_1)$ for solving $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$.

8. Write down Adam-Bashforth predictor and corrector formulae.

9. Write down the finite difference scheme for solving $\frac{d^2y}{dx^2} = x + y,\ y(0) = 0 = y(1)$ with $h = 0.5$.

10. State the explicit formula for the one dimensional wave equation $u_{tt} = \alpha^2 u_{xx}$ with $1 - \lambda^2\alpha^2 = 0$, where $\lambda = \frac{k}{h}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the smallest positive root of $x^3 - 2x - 5 = 0$ by the fixed point iteration method, correct to three decimal places. (6)

(ii) Find all eigenvalues and the corresponding eigenvectors of a matrix $A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$ by Jacobi's method. (10)

Or

(b) (i) Find, by Power method, the largest eigenvalue and the corresponding eigenvector of a matrix $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ starting with initial vector $X^{(0)} = (1\ \ 0\ \ 0)^T$. (8)

(ii) Solve the following system of equations by Gauss-Seidel method, correct to three decimal places :

$28x + 4y - z = 32;\ x + 3y + 10z = 24$ and $2x + 17y + 4z = 35$. (8)

[perform 4 iterations in each above 4 questions]

12. (a) (i) Find the interpolation polynomial $f(x)$ by Lagrange's formula and hence find $f(3)$ for (0, 2), (1, 3), (2,12) and (5, 147). (8)

(ii) Find the interpolation polynomial $f(x)$ by using Newton's forward interpolation formula and hence find the value of $f(5)$ from the following data : (8)

x :	4	6	8	10
$f(x)$:	1	3	8	16

Or

(b) Find the cubic spline approximation for the function given below : (16)

x :	0	1	2	3
$f(x)$:	1	2	33	244

Assume that $M(0) = 0 = M(3)$. Hence find the value of $f(2.5)$.

13. (a) (i) Find the first derivative of y with respect to x at $x = 10$ from the following data (6)

x :	3	5	7	9	11
y :	31	43	57	41	27

(ii) Using Gaussian three point formula, evaluate $\int_0^2 \frac{(x+1)^2}{1+(x+1)^4}\,dx$. (10)

Or

(b) (i) The following data give the corresponding values for pressure (p) and specific volume (v) of a superheated steam. Find the rate of change of pressure with respect to volume when $v = 2$. (8)

v :	2	4	6	8	10
p :	105	42.7	25.3	16.7	13.0

(ii) Using Simpson's one-third rule, evaluate $\int_0^{0.6} e^{-x^2}\,dx$ correct to three decimal places by step-size = 0.1. (8)

14. (a) (i) Find the values of y at $x = 0.1$ given that $\frac{dy}{dx} = x^2 - y,\ y(0) = 1$ by Taylor's series method. (8)

(ii) Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2},\ y(0) = 1$ by Runge-Kutta method of fourth order to find $y(0.2)$ with step size = 0.2. (8)

Or