(b) (i) Solve $u_{xx} = 2u_t$, given $u(0,t) = 0 = u(4,t)$, $u(x,0) = x(4-x)$. Assume $h = 1$ find the values of u up to $t = 5$ by Bender-Schmidt's method. (8)

(ii) Solve numerically $4u_{xx} = u_{tt}$ with the boundary conditions $u(0, t) = 0$, $u(4, t) = 0$ and the initial conditions $u_t(x, 0) = 0$, $u(x,0) = x(4-x)$ taking $h = 1$ for 4 time steps. (8)

———————

21/12/23 - FN.

Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 70860

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Fourth/Fifth/Sixth Semester

Civil Engineering

MA 8491 – NUMERICAL METHODS

(Common to Aeronautical Engineering/Aerospace Engineering/ Agriculture Engineering/Civil Engineering/Electrical and Electronics Engineering/ Electronics and Instrumentation Engineering/Instrumentation and Control Engineering/Manufacturing Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Biotechnology and Biochemical Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Plastic Technology/Polymer Technology/Textile technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the sufficient condition for the convergence of Gauss Seidel method?

2. What is the geometrical meaning of Newton's method?

3. What is interpolation?

4. Find $\Delta(xe^x)$.

5. Evaluate $\int_{-1}^{1}(3x^2 + 5x^4)dx$ by Gaussian three point formula.

6. Why trapezoidal rule is called so?

7. Given $y' = -y$, $y(0) = 1$ find the value of y at $x = 0.01$ using Euler method.

8. Compare Taylor Series and Runge-Kutta method of fourth order.

9. Write the finite difference scheme of the differential equation $y'' + 2y = 0$.

10. Write down the Laplace and Poisson equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to six decimal places. (8)

(ii) Apply Gauss-elimination method to obtain the solution of the system (8)

$3x + 4y + 5z = 18$, $2x - y + 8z = 13$ and $5x - 2y + 7z = 20$

Or

(b) (i) Solve the following system of equations $10x - 5y - 2z = 3$, $4x - 10y + 3z = -3$, $x + 6y + 10z = -3$ by Gauss-Seidel method. (8)

(ii) Find the largest eigen value of $\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$ by power method. (8)

12. (a) (i) Prove that the following :

(1) $E\nabla = \Delta = \nabla E$

(2) $hD = \log(1 + \Delta) = -\log(1 - \nabla)$. (8)

(ii) Find the values of y at $x = 21$ and $x = 28$ from the following data. (8)

X	20	23	26	29
Y	0.3420	0.3907	0.4384	0.4848

Or

(b) (i) Form the divided difference table for the data given below. (8)

X:	–2	0	3	5	7	8
Y:	–792	108	–72	48	–144	–252

(ii) Using Lagrange's formula, fit a polynomial for the given data below and hence find $y(x = 1)$. (8)

X:	–1	0	2	3
Y:	–8	3	1	12

13. (a) (i) The population of a certain town is given below. Find the rate of growth of population in 1931 and 1961. (8)

X (Year)	1931	1941	1951	1961	1971
Y (Population in 1000)	40.62	60.80	79.95	103.56	132.65

(ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Gaussian two point formula. (8)

Or

(b) Evaluate $\int_1^{1.4}\int_2^{2.4} \frac{dx\,dy}{xy}$ using Trapezoidal and Simpson's rules. Verify the result by actual integration. (16)

14. (a) (i) Use Taylor series method to find y at $x = 0.1$ and $x = 0.2$, given

$\frac{dy}{dx} = x^2 - y$, $y(0) = 1$. (8)

(ii) Apply Milne's method, find $y(2)$ if $y(x)$ is a solution of $\frac{dy}{dx} = \frac{1}{2}(x + y)$, $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$, $y(1.5) = 4.968$. (8)

Or

(b) (i) Using Runge-Kutta method, find $y(0.1)$ given that $\frac{dy}{dx} = x + y$, $y(0) = 1$, $h = 0.1$. (8)

(ii) Using Adam's method, find $y(0.4)$, given that $\frac{dy}{dx} = \frac{xy}{2}$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$. (8)

15. (a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ over the square mesh of side four units satisfying the following boundary conditions :

(i) $u(0, y) = 0$, $0 \le y \le 4$

(ii) $u(4, y) = 12 + y, 0 \le y \le 4$

(iii) $u(x, 0) = 3x$, $0 \le x \le 4$

(iv) $u(x,4) = x^2$, $0 \le x \le 4$. (16)

Or