Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 41520

B.E./B.Tech. DEGREE EXAMINATIONS, JANUARY 2022.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to All Branches (Except : Marine Engineering)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $2, -1, -3$ are the eigenvalues of a matrix "A", then find the eigenvalues of the matrix $A^2 - 2I$.

2. Write down the matrix for the following quadratic form:

$2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1 x_2 - 6x_1 x_3 + 6x_2 x_3$.

3. Find the domain of the function $f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$.

4. Evaluate the limit $\lim_{x \to 1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$.

5. If $u = x^3 + y^3$ where $x = a\cos t$ and $y = b\sin t$ then find $\dfrac{du}{dt}$.

6. If $u = \dfrac{2x - y}{2}$ and $v = \dfrac{y}{z}$ then find $\dfrac{\partial(u, v)}{\partial(x, y)}$.

7. Given that $\int_0^{10} f(x)\, dx = 17$ and $\int_0^8 f(x)\, dx = 12$ then find $\int_8^{10} f(x)\, dx$.

8. Determine whether the integral $\int_0^{\infty} \frac{dx}{x^2+4}$ is convergent or divergent.

9. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} d\theta \; dr$.

10. Evaluate $\int_0^1 \int_0^2 \int_0^3 \left[x \; y^2 \; z\right] dx \; dy \; dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}.$$ (8)

(ii) Using Cayley — Hamilton theorem find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$ (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1 x_2 + 2x_1 x_3 - 2x_2 x_3$ to canonical form through an orthogonal transformation. Also find its nature, rank, index and signature. (16)

12. (a) (i) If $x^2 + y^2 = 25$, then find $\frac{dy}{dx}$ and also find an equation of the tangent line to the curve $x^2 + y^2 = 25$ at the point $(3, 4)$. (8)

(ii) If $f(x) = xe^x$ then find $f'(x)$. Also find the n-th derivative $f^n(x)$. (8)

Or

(b) (i) Differentiate the function $f(x) = \frac{\sec x}{1 + \tan x}$. For what values of x, the graph of $f(x)$ has a horizontal tangent? (8)

(ii) Find the absolute maximum and absolute minimum values of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$. (8)

13. (a) (i) If $u = \log[\tan x + \tan y + \tan z]$ then find the value of $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z}$. (8)

(ii) Find the minimum value of $f(x, y) = x^2 + y^2 + 6x + 12$. (8)

Or

(b) (i) Expand $f(x, y) = e^x \sin y$ in terms of powers of "x" and "y" up to third degree terms by using Taylor's series. (8)

(ii) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (8)

14. (a) (i) Evaluate $\int \cos^n x \, dx$ by using integration by parts. (8)

(ii) Evaluate $\int \frac{dx}{\sqrt{3x - x^2 - 2}}$. (8)

Or

(b) (i) Evaluate $\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} dx$ by using the method of partial fractions. (8)

(ii) Evaluate $\int \frac{2x + 3}{x^2 + x + 1} dx$. (8)

15. (a) (i) Evaluate $\iint [x\, y]\, dx\, dy$ where the region of integration is bounded by the lines x-axis, $x = 2a$ and the curve $x^2 = 4ay$. (8)

(ii) Change the order of the integration in $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} [x\, y]\, dy\, dx$ and hence evaluate it. (8)

Or

(b) (i) Evaluate $\int_0^a \int_y^a \left[\frac{x}{x^2 + y^2}\right] dx\, dy$ by changing into polar coordinates. (8)

(ii) Evaluate $\int_0^{2a} \int_0^x \int_y^x [x\ y\ z]\, dz\, dy\, dx$. (8)
