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Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 60043

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ then find the eigen values of A^{-1}.

2. Prove that $x^2 - y^2 + 4z^2 + 4xy + 2yz + 6xz$ is indefinite.

3. Evaluate : $\lim_{x \to 5} (2x^2 - 3x + 4)$.

4. Find the domain of the function $f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$.

5. Prove $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$ if $f = x^3 + y^3 + z^3 + 3xyz$.

6. If $z = x^2 + y^2$, and $x = t^2$, $y = 2at$, find $\dfrac{dz}{dt}$.

7. Evaluate : $\int_0^{\pi/2} \sin^6 x \, dx$.

8. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

9. Evaluate : $\int_1^3 \int_1^2 xy^2 \, dxdy$.

10. Evaluate : $\int_0^3 \int_0^2 \int_0^1 xyz \, dxdy \, dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^4 if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form through an orthogonal reduction. (16)

12. (a) (i) For what values of a and b, is $f(x) = \begin{cases} -2, & x \le -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$ continuous at every x? (8)

(ii) Find the differential coefficients of $\frac{(a-x)^2 (b-x)^3}{(c-2x)^3}$. (8)

Or

(b) (i) Evaluate (1) $\frac{d}{dx}(3x^5 \log x)$ and (2) $\frac{d}{dx}\left(\frac{x^3}{3x-2}\right)$. (4+4)

(ii) Find the maximum and minimum values of $2x^3 - 3x^2 - 36x + 10$. (8)

13. (a) (i) If $x = u \cos v$ and $y = u \sin v$, prove that $\frac{\partial(x,y)}{\partial(u,v)} \times \frac{\partial(u,v)}{\partial(x,y)} = 1$. (8)

(ii) Obtain the Taylor's series expansion of $e^x \log(1+y)$ at the orign. (8)

Or

(b) (i) If $u = \log\left(\frac{x^5 + y^5}{x^3 + y^3}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$. (8)

(ii) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. (8)

14. (a) (i) Evaluate $\int \frac{x + \sin x}{1 + \cos x} dx$. (8)

(ii) Use partial fraction technique, evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx$. (8)

Or

(b) (i) Evaluate $\int_0^{\pi/4} \log(1 + \tan\theta) d\theta$. (8)

(ii) Find the mass M and the center of mass $\bar{x}$ of a rod lying on the x-axis over the interval $[1,2]$ whose density function is given by $\delta(x) = 2 + 3x^2$. (8)

15. (a) (i) Change the order of the integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ and evaluate the same. (8)

(ii) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$. (8)

Or

(b) (i) Using polar coordinates, evaluate $\iint_R e^{x^2+y^2} \, dydx$, where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$. (8)

(ii) Calculate the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$. (8)