19/09/23 - FN

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 30234

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

First Semester

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If two eigen values of the matrix $A=\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal to 1 each, find the eigen value of A^{-1}.

2. Write the uses of Cayley-Hamilton Theorem.

3. If $y = x\log\left(\frac{x-1}{x+1}\right)$, then find $\frac{dy}{dx}$.

4. Find the point of inflection of $f(x)=x^3-9x^2+7x-6$.

5. Write Euler's theorem on homogeneous functions.

6. If $x=r\cos\theta$, $y=r\sin\theta$, find $\frac{\partial(x, y)}{\partial(r,\theta)}$.

7. Evaluate $\int \theta\cos\theta\, d\theta$ using integration by parts.

8. Find the value of $\int_0^{\pi/2} \sin^6 x\, dx$.

9. Evaluate $\int_0^1\int_0^x dy\, dx$.

10. Transform the double integral $\int_0^2\int_y^2 \frac{x\,dx\,dy}{x^2+y^2}$ into polar coordinates.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $A=\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $2x_1x_2 - 2x_2x_3 + 2x_3x_1$ into the canonical form and hence find its nature. (16)

12. (a) (i) Find the values of a and b that make f continuous on $(-\infty, \infty)$ if

$$f(x)=\begin{cases}\dfrac{x^3-8}{x-2}, & \text{if } x<2\\ ax^2-bx+3, & \text{if } 2\le x<3\\ 2x-a+b, & \text{if } x\ge 3\end{cases}$$ (8)

(ii) Find $\dfrac{dy}{dx}$ if $y=x^2e^{2x}(x^2+1)^4$. (4)

(iii) If $x^y=y^x$, Prove that $\dfrac{dy}{dx}=\dfrac{y(y-x\log y)}{x(x-y\log x)}$ using implicit differentiation. (4)

Or

(b) (i) Show that $\sin x(1+\cos x)$ is maximum when $x=\pi/3$. (6)

(ii) A window has the form of a rectangle surmounted by a semicircle. If the perimeter is 40 ft., find its dimensions so that greatest amount of light may be admitted. (10)

13. (a) (i) Given the transformations $u=e^x\cos y$ and $v=e^x\sin y$ and that f is a function of u and v and also of x and y, prove that $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}=(u^2+v^2)\left(\dfrac{\partial^2 f}{\partial u^2}+\dfrac{\partial^2 f}{\partial v^2}\right)$. (8)

(ii) Expand $e^x\log(1+y)$ in powers of x and y up to terms of third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y)x^4+y^4-2x^2+4xy-2y^2$. (8)

(ii) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box, that requires the least material for its construction. (8)

14. (a) (i) Evaluate $\int\dfrac{3x+1}{(x-1)^2(x+3)}dx$ by applying partial fraction on the integrand; (6)

(ii) Evaluate $\int_0^{\pi/2}\log\sin x\,dx$ and hence find the value of $\int_0^1\dfrac{\sin^{-1}x}{x}dx$. (10)

Or

(b) (i) Evaluate $\int\dfrac{\sqrt{9-x^2}}{x^2}dx$ using trigonometric substitution. (6)

(ii) Determine whether the integral $\int_1^\infty\dfrac{1}{x}dx$ is convergent or divergent. (4)

(iii) Find the volume of the reel shaped solid formed by the revolution about the y-axis, of the part of the parabola $y^2=4ax$ cut off by its latusrectum. (6)

15. (a) (i) Find the area between the curves $y^2=4x$ and $x^2=4y$. (8)

(ii) Change the order of integration in $\int_0^\infty\int_0^y ye^{-y^2/x}\,dx\,dy$ and then evaluate it. (8)

Or

(b) (i) Find the volume of the sphere of radius 'a'. (8)

(ii) Find the moment of inertia of the area bounded by the curve $r^2=a^2\cos 2\theta$ about its axis. (8)