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Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 21272

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the eigenvalues of A^{-1} and A^2 if $A = \begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$.

2. State Cayley-Hamilton theorem.

3. Sketch the graph of the function $f(x) = \begin{cases} x^2 & if\ -2 \le x \le 0 \\ 2-x & if\ 0 < x \le 2 \end{cases}$.

4. The equation of motion of a particle is given by $s = 2t^3 - 5t^2 + 3t + 4$ where s is measured in meters and t in seconds. Find the velocity and acceleration as functions of time.

5. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

6. Write any two properties of Jacobians.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^9 x\,dx$.

8. Prove that the integral $\int_1^{\infty} \frac{1}{x} dx$ is divergent.

9. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$.

10. Find the area of a circle $x^2+y^2=a^2$ using polar coordinates in double integrals.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of $A=\begin{pmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{pmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^{-1} if $A=\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4zx$ into the canonical form and hence find its rank, index, signature and nature. (16)

12. (a) (i) Let $f(x)=\begin{cases}\sqrt{-x} & if\ x<0\\ 3-x & if\ 0\le x\le 3\\ (x-3)^2 & if\ x>3\end{cases}$. Evaluate each of the following limits, if they exist.

(1) $\lim_{x\to 0^-} f(x)$

(2) $\lim_{x\to 0^+} f(x)$

(3) $\lim_{x\to 3^-} f(x)$

(4) $\lim_{x\to 3^+} f(x)$

(5) $\lim_{x\to 0} f(x)$

(6) $\lim_{x\to 3} f(x)$

Also, find where $f(x)$ is continuous. (8)

(ii) Find the n^{th} derivative of $f(x)=xe^x$. (4)

(iii) Differentiate $F(t)=\dfrac{t^2}{\sqrt{t^3+1}}$. (4)

Or

(b) (i) Use logarithmic differentiation to differentiate $y=\dfrac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$. (8)

(ii) Discuss the curve $f(x)=x^4-4x^3$ for points of inflection, and local maxima and minima. (8)

13. (a) (i) Given the transformations $u=e^x\cos y$ and $v=e^x\sin y$ and that f is a function of u and v and also of x and y, prove that

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=\left(u^2+v^2\right)\left(\frac{\partial^2 f}{\partial u^2}+\frac{\partial^2 f}{\partial v^2}\right)$$ (8)

(ii) Expand $e^x\cos y$ in a series of powers of x and y as far as the terms of the third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x,y)=x^3+y^3-12x-3y+20$. (8)

(ii) A rectangular box, open at the top is constructed so as to have a volume of 108 cubic meters. Find the dimensions of the box that requires the least material for its construction. (8)

14. (a) (i) Find a reduction formula for $\int e^{ax}\sin^n x\,dx$. (8)

(ii) Integrate the following : $\int \dfrac{x^4-2x^2+4x+1}{x^3-x^2-x+1}dx$. (8)

Or

(b) (i) Evaluate $\int\sqrt{\dfrac{1-x}{1+x}}\,dx$. (8)

(ii) Find the centre of mass of a semicircular plate of radius r. (8)

15. (a) (i) Change the order of integration in $\int_0^4\int_{x^2/4}^{2\sqrt{x}} xy\,dy\,dx$ and then evaluate it. (8)

(ii) Find the area enclosed by the curves $y=2x-x^2$ and $x-y=0$. (8)

Or

(b) (i) Find the volume of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (8)

(ii) Find the moment of inertia of a hollow sphere about a diameter, given that its internal and external radii are 4 meters and 5 meters respectively. (8)