

Reg. No. :

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

|                                    |
|------------------------------------|
| <b>Question Paper Code : 70138</b> |
|------------------------------------|

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Mechanical Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Aerospace Engineering/  
Automobile Engineering/Biomedical Engineering/  
Civil Engineering/Manufacturing Engineering/Marine Engineering/  
Materials Science and Engineering/Mechanical Engineering (Sandwich)/  
Mechanical and Automation Engineering/Mechatronics Engineering/  
Medical Electronics/ Petrochemical Engineering/Production Engineering/  
Robotics and Automation/Safety and Fire Engineering/Bio Technology/  
Biotechnology and Biochemical Engineering/Food Technology/  
Petrochemical Technology/ Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the complete solution of the PDE  $p^2 + q^2 = 4pq$ .
2. Find the complementary function of the PDE  $(D^3 - 3DD'^2 + 2D'^3)z = e^{2x-y}$ .
3. Identify the given  $f(x)$  is an even or odd function. Also sketch its graph.

$$f(x) = \begin{cases} \pi + x & -\pi \leq x \leq -\frac{\pi}{2} \\ -x & -\frac{\pi}{2} \leq x \leq 0 \\ x & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

4. State Parseval's identity in Fourier series.

5. Write down the appropriate solution of the one dimensional heat flow equation. How is it chosen?
6. The ends A and B of a rod 30 cm long, have their temperature kept at  $10^\circ\text{C}$  and  $100^\circ\text{C}$  respectively. Then obtain the steady state temperature.
7. What are the sufficient conditions for the existence of Fourier transform of a function  $f(x)$ ?
8. Obtain the Fourier cosine transform of  $\frac{1}{2^t}$ .
9. Find the inverse Z transform of  $\frac{z}{(z-1)^2}$ .
10. State final value theorem in Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve :  $(x-2z)p + (2z-y)q = y-x$ . (8)  
(ii) Solve :  $(D^3 + D^2D' - 4DD'^2 - 4D'^3)z = \cos(2x+y)$ . (8)  
Or  
(b) (i) Solve the PDE  $2z + p^2 + qy + 2y^2 = 0$ . (8)  
(ii) Solve  $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x+2y)$ . (8)
12. (a) (i) Obtain the Fourier series of periodicity  $2\pi$  for  $f(x) = e^x$  in the interval  $0 < x < 2\pi$ . (8)  
(ii) Obtain the half range Fourier cosine series of  $f(x) = x(l-x)$  in  $(0, l)$ . (8)

Or

- (b) The following table gives the variations of periodic current over a period.
- |         |      |      |      |      |       |       |      |
|---------|------|------|------|------|-------|-------|------|
| t sec : | 0    | T/6  | T/3  | T/2  | 2T/3  | 5T/6  | T    |
| A amp : | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. (Harmonic Analysis). (16)

13. (a) A tightly stretched string of length  $2l$  is fastened at both ends. The midpoint of the string is displaced by a distance 'b' transversely and the string is released from rest in this position. Find the displacement  $y$  at any distance  $x$  from one end at any time  $t$ . (16)

Or

- (b) An infinitely long metal plate in the form of an area is enclosed between the lines  $y=0$  and  $y=\pi$  for positive values of  $x$ . The temperature is zero along the edges  $y=0$  and  $y=\pi$  and the edge at infinity. If the edge  $x=0$  is kept at temperature 'ky', find the steady state temperature at any point in the plate. (16)

14. (a) (i) Find the Fourier transform of  $f(x)$  given by  $f(x) = \begin{cases} 1, & \text{for } |x| \leq a \\ 0, & \text{for } |x| > a \end{cases}$ . (8)  
(ii) Find the Fourier sine transform of  $f(x) = e^{-ax}$ ,  $a > 0$  and hence find  $F_C(xe^{-ax})$ . (8)

Or

- (b) (i) Using Parseval's identity for Fourier transforms, evaluate  $\int_0^\infty \frac{ds}{(a^2+s^2)(b^2+s^2)}$ . (8)  
(ii) Find the Fourier cosine transform of  $f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2-x & ; 1 < x < 2 \\ 0 & ; x > 2 \end{cases}$ . (8)

15. (a) Using Z transform, solve the difference equation  $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$  with  $u_0 = 0$ ,  $u_1 = 1$ . (16)

Or

- (b) State and prove convolution theorem in Z transforms and use it to find  $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$ . (16)