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**Question Paper Code : 30240**

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Aeronautical Engineering

MA 3351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aerospace Engineering/Automobile Engineering/Biomedical Engineering/Civil Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/ Mechanical Engineering (Sandwich)/ Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/Bio Technology/Biotechnology and Biochemical Engineering/Food Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form the PDE by eliminating the arbitrary constants '$a$ and $b$' from the relation $z = (x+a)\,(y+b)$.

2. Find the PDE corresponding to the complementary function $z = f_1(y+x) + x\,f_2(y+x) + f_3(y-x) + x\,f_4(y-x)$.

3. If $f(x)$ is defined in $(-\pi, \pi)$ and if $f(x) = x+1$ in $(0, \pi)$, then find $f(x)$ in $(-\pi, 0)$ if

   (a) $f(x)$ is odd

   (b) $f(x)$ is even.

4. Determine the value of $b_{25}$ while expanding the function $f(x) = \begin{cases} 1+\dfrac{2x}{l}; & -l \le x \le 0 \\ 1-\dfrac{2x}{l}; & 0 \le x \le l \end{cases}$ as a Fourier series.

5. A tightly stretched string of length '$2L$' is fastened at both ends. The midpoint of the string is displaced to a distance '$b$' and released from rest in this position. Write the boundary conditions.

6. The ends $A$ and $B$ of a rod 100 cm long, have their temperatures kept at 10°C and 100°C respectively. Then find the steady state temperature distribution function.

7. Obtain the Fourier transform of $f(x) = \begin{cases} 1; & for\ |x| \le 1 \\ 0; & for\ |x| > 1. \end{cases}$

8. State Convolution theorem for Fourier transforms.

9. Find the inverse $Z$ transform of the unit impulse sequence $\delta(n) = \begin{cases} 1; & for\ n = 0 \\ 0; & for\ n \ne 0. \end{cases}$

10. State Initial value theorem in $Z$ transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $(x - 2z)p + (2z - y)q = y - x$ (8)

(ii) Solve : $(D^2 + DD' - 6D'^2)z = \cos(2x + y)$. (8)

Or

(b) (i) Solve : $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. (8)

(ii) Obtain the general solution of $(D^2 - 2DD' + D'^2)z = \sin x$. (8)

12. (a) Obtain the Fourier series expansion of $f(x) = 2x - x^2$ in the interval $0 < x < 3$. (16)

Or

(b) The displacement $y$ of a part of a mechanism is tabulated with corresponding angular movement $x°$ of the crank. Express $y$ as a Fourier series neglecting the harmonics above the third. (16)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x°$ : | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 |
| $y$ : | 1.80 | 1.10 | 0.30 | 0.16 | 1.50 | 1.30 | 2.16 | 1.25 | 1.30 | 1.52 | 1.76 | 2.00 |



13. (a) A tightly stretched string of length $l$ is fastened at both ends. Initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y$ at any distance $x$ from one end at any time $t$. (16)

Or

(b) A bar with 100 cm long, with insulated sides, has its ends kept at 0°C and 100°C respectively until steady state conditions prevail. The two ends are then suddenly insulated and kept so. Then, find the temperature distribution function. (16)

14. (a) (i) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 - x^2, & for\ |x| \le 1 \\ 0, & for\ |x| > 1. \end{cases}$ Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$. (10)

(ii) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that $F\{f(x - a)\} = e^{isa} F(s)$. (6)

Or

(b) Find the Fourier transform of $f(x) = e^{-a^2x^2}$, $a > 0$. Hence deduce that $e^{\frac{-x^2}{2}}$ is self reciprocal in respect of Fourier transform. (16)

15. (a) Using $Z$ transform, solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$. (16)

Or

(b) State and prove the convolution theorem in $Z$ transforms and apply it to find $Z^{-1}\left\{\frac{z^2}{(z-2)(z-3)}\right\}$. (16)

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