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**Question Paper Code : 50400**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Seventh/Eighth Semester

Computer Science and Engineering

CS 6702 – GRAPH THEORY AND APPLICATIONS

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions

PART – A

(10×2=20 Marks)

1. Determine the number of vertices for a graph  $G$ , which has 15 edges and each vertex has degree 6. Is the graph  $G$  be a simple graph?
2. Suppose  $G$  is a finite cycle-free connected graph with at least one edge. Show that  $G$  has at least two vertices of degree 1.
3. In a tree, every vertex is a cut-vertex. Justify the claim.
4. A simple planar graph to which no edge can be added without destroying its planarity (while keeping the graph simple) is a maximal planar graph. Prove that every region in a maximal planar graph is a triangle.
5. Prove that a graph of  $n$  vertices is a complete graph iff its chromatic polynomial is  $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1)$ .
6. Define the two types of connectedness in digraphs. Give examples.
7. THALASSEMIA is a genetic blood disorder. How many ways can the letters in THALASSEMIA be arranged so that all three A's together?
8. Determine the number of positive integers  $n$ ,  $1 \leq n \leq 100$ , that are not divisible by 3 or 7.
9. Find the coefficient of  $x^6$  in  $(3 - 5x)^{-8}$ .
10. The number of virus affected files in a system is 500 (approximately) and this doubles every four hours. Using a recurrence relation, determine the number of virus affected files in the system after one day.

- ii) Find the value of sum if the given program segment is executed.

```

main ()
{
    int inc = 0, sum = 0;
    int i, j, k;
    for (i=1; i≤10; i++)
        for (j=1; j≤i; j++)
            for (k=1; k≤j; k++)
                {
                    inc = inc + 1;
                    sum = sum + inc;
                }
}
  
```

(10)

(OR)

- b) i) Determine the coefficient of  $x^9y^6$  in the expansion of  $(4y - x)^{15}$ . (4)
- ii) How many integer solutions are possible for  $x_1 + x_2 + x_3 + x_4 + x_5 < 40$  where  $x_i \geq -3$ ,  $1 \leq i \leq 5$ . (6)
- iii) In a survey of the chewing gum tastes of a group of baseball players, it was found that 22 liked juicy fruit, 25 liked spearmint, 39 like bubble gum, 9 like both spearmint and juicy fruit, 17 liked juicy fruit and bubble gum, 20 liked spearmint and bubble gum, 6 liked all three and 4 liked none of these. How many baseball players were surveyed? (6)
15. a) i) Two cases of soft drinks, 24 bottles of one type and 24 bottles of another, are distributed among five surveyors who are conducting taste tests. In how many ways can the 48 bottles be distributed so that each surveyor gets at least two bottles of each type? And in how many ways can they be distributed so that each surveyor gets at least two bottles of one type and three of other type? Use generating function. (12)
- ii) Find all partitions of integer 6 and find the number of partitions with distinct summands. (4)

(OR)

- b) i) A person invests Rs. 50,000 at 6% interest compounded annually.
- 1) Find the amount at the end of 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> year.
  - 2) Write the general explicit formula.
  - 3) How long will it take to double the investment? Use recurrence relation. (10)
- ii) Derive an explicit formula for the Fibonacci sequence using recurrence relation. (6)



11. a) i) The Figure 1 represents a floor plan with the doors between the rooms and the outside indicated. The real estate agent would like to tour the house, starting and ending outside, by going through each door exactly once. What is the fewest number of doors that should be added, and where should they be placed in order to make this tour possible? Give reasons for your answer.

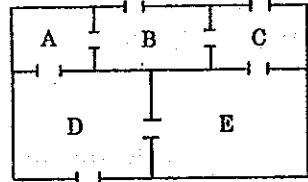


Fig. 1

- ii) Define closed-walk, open-walk, path and circuit. Take a graph of your choice and give an example to each one.

(OR)

- b) i) Nine members of committee have their dinner in round table. If no member sits near to the same neighbour more than once, how many days can this arrangement possible? Write all possible arrangements.

- ii) State four properties of a tree graph and prove them.

12. a) i) Show that starting from any spanning tree of a graph G, every other spanning tree of G can be obtained by successive cyclic interchanges.

- ii) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets.

- iii) Define edge vertex connectivity and edge connectivity. Give the relation between them.

(OR)

- b) i) Show, by drawing the graphs, that two graphs with the same rank and the same nullity need not be 2-isomorphic.

- ii) State Kuratowski's Theorem and use it in order to prove the graph in Fig. 2 is non-planar.

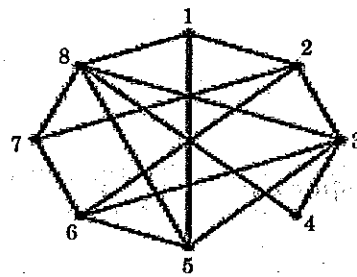


Fig. 2

(8)

(8)

(8)

(8)

(6)

(6)

(4)

(4)

(8)



- iii) State minimum cut maximum flow theorem. Using it calculate the maximum flow between the nodes D and E in the graph (Fig. 3). The number on a line represents the capacity.

(4)

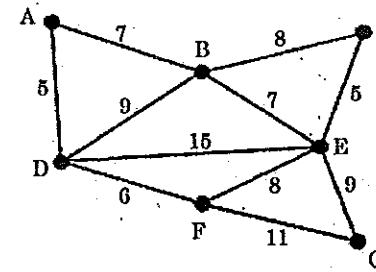


Fig. 3

13. a) i) Obtain the chromatic polynomial of the graph G in Fig. 4 using the theorem.  $P_n(\lambda)$  of  $G = P_n(\lambda)$  of  $G' + P_n(\lambda)$  of  $G''$ .

(8)

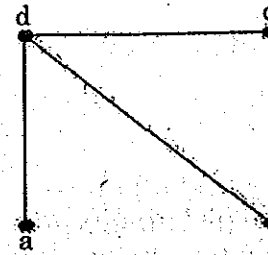


Fig. 4

- ii) State and prove five-color theorem.

(8)

(OR)

- b) i) Define the following and give one example to each :

Complete Matching

Minimal Covering

Balanced Digraph

Strongly Connected Digraph

Fragment in a digraph.

(10)

- ii) Prove that a digraph G is an Euler digraph if and only if G is connected and is balanced. Draw an example Euler digraph of 6 vertices.

(6)

14. a) i) There are five students in a group and their roll numbers are S1, S2, S3, S4, S5 and S6. They are given with five assignments numbered 1 to 6. Each has to solve one assignment. How many ways the assignments can be distributed such that a student is not getting an assignment number same as his roll number?

(6)