ii) Derive the steady state system size probabilities for a M/M/1/N/FCFS queueing model and hence obtain the mean number of customers in the queue. (8)

(OR)

b) i) Derive the steady-state system-size probabilities for a M/M/C/∞ FCFS queueing model and hence obtain the mean number of customers in the system. (8)

ii) Patients arrive at a clinic according to a Poisson process at a rate of 3 patients per hour. The waiting room cannot accommodate more than 6 patients. Examination time per patient is exponentially distributed random variable with rate of 4 per hour.

1) Find the effective arrival rate at the clinic.

2) What is the probability that an arriving patient will not wait ?

3) What is the expected waiting time W_s in the system ? (8)

15. a) Discuss an M/G/1/∞ FCFS queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula for the system size. Deduce also the mean number of customers in the system for M/M/1/∞ FCFS queueing model from the P-K mean value formula. (16)

(OR)

b) Derive the system of differential difference equations for the joint probabilities of the system size of two-station tandem queueing system. Under the steady-state conditions, determine the steady-state probabilities of the system size and obtain 1) Expected number of customers in the system, 2) The mean waiting time in the system. (16)

Reg. No. :

Question Paper Code : 90342

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Fourth Semester

Computer Science and Engineering

MA8402 – PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. A bag contains 8 white and 4 black balls. If 5 balls are drawn at random, what is the probability that 3 are white and 2 are black ?

2. Let $M_X(t) = \frac{1}{1-t}$, $|t| < 1$, be the moment generating function of a R.V. X. Find E(X) and $E(X^2)$.

3. If $f(x, y) = e^{-(x+y)}$, $x \geq 0$, $y \geq 0$, is the joint probability density function of (X, Y), Find $P(X + Y \leq 1)$.

4. Let X and Y be independent R.Vs with Var(X) = 9 and Var(Y) = 3. What is Var(4X – 2Y + 6) ?

5. Define : Markov process.

6. Let $\{X_n ; n \geq 0\}$ be a Markov chain having state space S = {1, 2} and one-step TPM $P = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$. Find the stationary probabilities of the Markov chain.

7. In an M/M/1/∞/FCFS queue, the service rate, $\mu = \frac{1}{3}$/ minute and waiting time in the queue $W_q = 3$ minute, compute the arrival rate, λ.

8. For a M/M/C/N/FCFS (C < N) queueing system, write the expressions for P_0 and P_N.

9. In an M/D/1 queueing system, an arrival rate of customers is 1/6 per minute and the server takes exactly 4 minutes to serve a customer. Calculate the mean number of customers in the system.

10. For an open Jackson queueing network, write the expression for traffic equations and stability condition of the system.

PART – B (5×16=80 Marks)

11. a) i) There are 3 boxes containing respectively, 1 white, 2 red, 3 black balls, 2 white, 3 red, 1 black balls; 3 white, 1 red, 2 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they came from second box ? (8)

ii) The p.d.f. of a continuous R.V. X is given by $f(x) = \begin{cases} \frac{x}{2}e^{-\frac{x}{2}}, & x > 0 \\ 0, & x \le 0 \end{cases}$. Obtain

1) C.D.F. of X, F(x)
2) $P(X > 1)$
3) $P(1 < X < 2)$
4) $E(X^2)$. (8)

(OR)

b) i) Let X be a binomial R.V with E(X) = 4 and Var(X) = 3. Find: (1) P(X = 5), (2) M.G.F. of X, $M_X(t)$, (3) $E(X^2 - 1)$, (4) Var $\left(-\frac{1}{2}X + 4\right)$. (8)

ii) A R.V. X is uniformly distributed on (–5, 15). Determine :

1) C.D.F. of X, F(x)
2) $P(X < 5/X > 0)$
3) $P(|X - 1| < 5)$
4) $E\left(e^{-\frac{X}{5}}\right)$. (8)

12. a) i) The joint p.d.f. of (X, Y) is given by $f(x, y) = \begin{cases} \frac{1}{240}, & 8.5 \le x \le 10.5, 120 \le y \le 240 \\ 0, & \text{otherwise} \end{cases}$.

Obtain

1) The marginal p.d.fs of X and Y.
2) E(X) and E(Y)
3) E(XY)
4) Are X and Y independent R.Vs ? Justify. (8)

ii) Let X and Y be two continuous R.Vs with joint p.d.f.

$f(x, y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$. Determine the joint p.d.f. of the R.Vs $U = X^2$ and $V = XY$ and hence obtain the marginal p.d.f. of U. (8)

(OR)

b) i) The joint p.d.f. of R.V (X, Y) is given as $f(x, y) = \begin{cases} Ce^{-(2x+3y)}, & 0 \le y \le x < \infty \\ 0, & \text{otherwise} \end{cases}$.

Find :

1) The value of C.
2) Are the R.Vs X and Y independent ? (8)

ii) Let X and Y be random variables such that E(X) = 1, E(Y) = 2, Var(X) = 6, Var(Y) = 9 and the correlation coefficient $\rho_{XY} = -\frac{2}{3}$. Calculate :

1) The covariance, Cov(X, Y), of X and Y
2) E(XY)
3) $E(X^2)$ and $E(Y^2)$. (8)

13. a) i) Consider a random process $X(t) = \cos(t + \phi)$, where ϕ is a R.V. such that $P(\phi = 0) = P(\phi = \pi) = \frac{1}{2}$. Determine 1) E(X(t)), 2) $E(X^2(t))$, 3) $R_{XX}(t, t + \tau)$. Is the process X(t) wide-sense stationary ? Justify. (8)

ii) State the postulates of a Poisson process $\{X(t) ; t \ge 0\}$ with parameter λ. Derive the system of differential difference equations and hence obtain the probability distribution, $P(X(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$, n = 0, 1, 2, ... (8)

(OR)

b) i) Let $\{X_n ; n \ge 0\}$ be a Markov chain having state space S = {1, 2, 3} with one-step TPM. $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$.

1) Draw a transition diagram.
2) Is the chain irreducible ? Explain.
3) Is the state – 2 ergodic ? Justify your answer. (8)

ii) Let X(t) and Y(t) be two independent Poisson processes with parameters λ_1 and λ_2 respectively. Obtain 1) P(X (t) + Y(t) = n), n = 0, 1, 2, ..., 2) P(X(t) – Y(t) = n), n = 0, ±1, ±2, ... (8)

14. a) i) A petrol station has one petrol pump. The cars arrive for service according to a Poisson process at a rate of 0.5 cars per minute and the service time for each car follows the exponential distribution with rate of 1 car per minute. compute :

1) The probability that the pump station is idle
2) The probability that 10 or more cars are in the system
3) The mean number, L_S of cars in the system.
4) The mean waiting time, W_q, in the queue and the mean waiting time, W_s, in the system. (8)