15. (a) (i) Derive P-K formula for the queuing model $(M/G/1):(\infty/GD)$. (8)

(ii) A repair facility shared by a large number of machines has 2 sequential stations with respective service rates of 2 per hour and 3 per hour. The cumulative failure rate of all the machine is 1 per hour. Assuming that the system behavior may be approximated by the 2-stage tandem queue. Find the average number of machines in the system, the average repair time including waiting time and the probability that both the service stations are idle. (8)

Or

(b) In a book shop, there are two sections, one for textbooks and the other for note books. Customers from outside arrive at the text book section at a Poisson rate of 4 per hour and the note book section at a Poisson rate of 3 per hour. The service rate of the text book section and notebook section are respectively 8 and 10 per hour. A customer upon completion of service at text book section is equally likely to go to the notebook section or to leave the book shop, whereas a customer upon completion of service at note book section will go to the text book section with probability 1/3 and will leave the bookshop otherwise. Find the joint steady state probability that there are 4 customers in text book section and 2 customers in the note book section. Find also the average number of customers in the book shop and the average waiting time of a customer in the shop. Assume that there is only one salesman in each section. (16)

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Reg. No. : [ ]

Question Paper Code : 90818

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Statistical Tables may be permitted

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If two fair dice are tossed, Find the probability of getting the sum of the outcomes of the two dice is equal to 7 or 11.

2. The CDF of the random variable X is defined by $F(X)=\begin{cases}0, & 0<x<2\\ A(x-2), & 2\le x<6\\ 1, & x\ge 6\end{cases}$, Find the value of A.

3. If X and Y are independent random variables with means 2 and 3 and variances 1 and 2 respectively, find the variance of $Z=2X-5Y$.

4. The two regression equations of two random variables X and Y are $4x-5y+33=0$ and $20x-9y=107$. Find the mean values of X and Y.

5. Define strict sense stationary process.

6. Write down the postulates of Poisson process.

7. What are the characteristics of a queuing system?

8. State Little's formula for single server finite capacity queuing system.

9. What is meant by bottle neck of the system in the queue networks?

10. State the flow balance equation for a two stage series queue with blocking.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A student buys 1000 integrated circuits (ICs) from supplier A, 2000 ICs from supplier B, and 3000 ICs from supplier C. He tested the ICs and found that the conditional probability of an IC being defective depends on the supplier from whom it was bought. Specifically, given that an IC came from supplier A, the probability that it is defective is 0.05; given that an IC came from supplier B, the probability that it is defective is 0.10; and given that an IC came from supplier C, the probability that it is defective is 0.10. If the ICs from the three suppliers are mixed together and one is selected at random, what is the probability that it is defective? (8)

(ii) The discrete random variable X has the following PMF $p(x) = \begin{cases} b, & x=0 \\ 2b, & x=1 \\ 3b, & x=2 \\ 0, & otherwise \end{cases}$ Find the value of b. Determine the values of $P[X<2], P[X \le 2]$ and $P[0<X<2]$. Also Determine the CDF of X. (8)

Or

(b) (i) Derive the moment generating function of Poisson distribution. Hence find its the first three moments about origin. (8)

(ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modeled by an N(5;16) normal random variable X. What is the probability that a randomly selected parcel weighs between 1 and 10 pounds? (8)

12. (a) (i) If the joint PDF of the random variables X and Y is $f(x) = \begin{cases} 25e^{-5y}, & 0<x<0.2,\ y>0 \\ 0, & otherwise \end{cases}$, find the marginal PDFs of X and Y. Are X and Y independent? (8)

(ii) Obtain the two regression lines from the following data: (8)

x:	62	64	65	69	70	71	72	74
y:	126	125	139	145	165	152	180	208

Or

(b) (i) Find the PDF of U, which is the sum of X and Y that are independent random variables with the following PDFs: $f(x) = \lambda e^{-\lambda x}, x \ge 0$ and $f(y) = \mu e^{-\mu y}, y \ge 0$ where $\lambda \ne \mu$. (8)

(ii) Assume that the random variable S_n is the sum of 48 independent experimental values of the random variable X whose PDF is given by $f(x) = \begin{cases} \frac{1}{3}, & 1 \le x \le 4 \\ 0, & otherwise \end{cases}$. Find the probability that S_n lies in the range $108 \le Sn \le 126$. (8)

13. (a) (i) A random process $X(t)$ is defined by $X(t) = A\cos t + B\sin t$, $-\infty < t < \infty$ where A and B are independent random variables each of which has a value -2 with probability 1/3 and a value 1 with probability 2/3. Show that $X(t)$ is a wide sense stationary process. (8)

(ii) If particles are emitted from a radioactive source at the rate 20 per hour, find the probability that exactly 5 particles are emitted during a 15 minutes period. (8)

Or

(b) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. (16)

14. (a) Customers arrive at a one-man barber shop according to a Poisson distribution with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber's chair. (i) What is the average number of customers in the barber shop? (ii) What is the average number of customers in the queue? (iii) What is the probability that a customer will not have to wait for a haircut? (iv) How much time can a customer expect to spend in the barber shop? (v) What is the average time a customer spends in the queue? (vi) What is the probability that the waiting time in the system is greater than 30 minutes? (vii) What is the probability that there are more than 3 customers in the system? (16)

Or

(b) At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river. Tankers arrive according to a Poisson process with a mean of 1 every 2 hours. It takes for an unloading crew, on the average, 10 hours to unload a tanker, the unloading time following an exponential distribution. Find (i) how many tankers are at the port on the average? (ii) how long does a tanker spend at the port on the average? (iii) what is the average arrival rate at the over flow facility? (16)