21-06-23 - FN

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 50835

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours Maximum : 100 marks

(Note : Statistical Table Need to given)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A fair coin was tossed two times. Given that the first toss resulted in heads, what is the probability that both tosses resulted in heads?

2. What are the limitations of Poisson distribution?

3. The joint pdf of a two-dimensional random variable (X,Y) is given by $f(x, y) = \begin{cases} ke^{-(x+y)}; & 0 \le x \le y,\ 0 \le y \le \infty \\ 0; & \text{otherwise} \end{cases}$. Find the value of '$k$'.

4. State Central limit theorem.

5. Is Poisson process stationary? Justify.

6. What do you mean by $X_n = 5$ and P_{ij} in Markov chain?

7. What is the steady state condition for M/M/s queueing model? Does the same steady state condition hold for a finite M/M/s/K queue? Why or why not?

8. What is the probability that a customer has to wait more than 30 minutes to get his service completed in a M/M/1 queueing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour?

9. State Jackson's theorem in open queuing network.

10. What do you mean by bottleneck of a network of queues?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Suppose you have two coins, one biased, one fair, but you don't know which coin is which. Coin 1 is biased. It comes up heads with probability 3/4, while coin 2 will flip heads with probability 1/2. Suppose you pick a coin at random and flip it. Let C_i denote the event that coin 'i' is picked. Let H and T denote the possible outcomes of the flip. Given that the outcome of the flip is head, what is the probability that you picked the biased coin? Given that the outcome is a tail, what is the probability that you picked the fair coin? (8)

(ii) If the random variable X takes the values 1,2,3 and 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$, find the probability distribution and cumulative distribution of X. (8)

Or

(b) (i) If the random variable 'K' is uniformly distributed over (0, 5). what is the probability that the roots of the equation $4x^2+4kx+(k+2)=0$ are real? (8)

(ii) In a class of 50, the average mark of students in a subject is 48 and standard deviation is 24. Find the number of students who got (1) above 50 (2) between 35 and 50. (8)

12. (a) (i) If the joint pdf of (X, Y) is $f(x, y)=6e^{-2x-3y}$; $x\geq 0$, $y\geq 0$, then find the marginal density function of X and conditional density function of Y given X. (8)

(ii) The lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1200 h and standard deviation 250 h. find the probability, using the central limit theorem, that the average lifetime of 60 bulbs exceeds 1250 h. (8)

Or

(b) (i) Let the joint pmf (probability mass function) of X and Y be given by $p(x, y)=\begin{cases}k(2x+y): & if\ x=1,2; y=1,2\\ 0: & \text{otherwise}\end{cases}$. Find the value of k, conditional pmf of Y given X, conditional pmf of X given Y. (8)

(ii) Find the coefficient of correlation between X and Y, using following data: (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

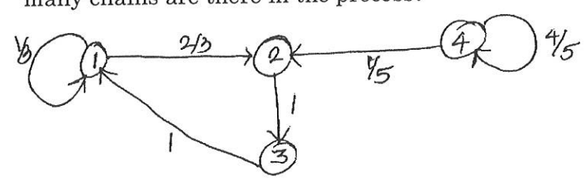
13. (a) (i) A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since the last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (8)

(ii) There are two white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. Let the state a_i of the system be the number of red marbles in A after 'i' changes. What is the probability that there are 2 red marbles in A after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn A? (8)

Or

(b) (i) State and prove Chapman-Kolmogorov equation. (8)

(ii) Consider the Markov chain with the state-transition diagram shown in the below figure. Identify the transient states, the recurrent states, and the periodic states with their periods. How many chains are there in the process? (8)



14. (a) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour,

(i) What fraction of the time all the typists will be busy? (4)

(ii) What is the average number of letters waiting to be typed? (4)

(iii) What is the average time a letter has to spend for waiting and for being typed? (4)

(iv) What is the probability that a letter will take longer than 20 min waiting to be typed and being typed? (4)

Or

(b) Explain Markovian Birth - Death process and obtain the expressions for steady state probabilities.

15. (a) Derive Pollaczek — Khintchine formula of an M/G/1 queue.

Or

(b) Write short notes on :

(i) Open queueing networks and (8)

(ii) Closed queueing networks. (8)

———————