14. (a) Explain Markovian Birth - Death process and obtain the expressions for steady state probabilities. (16)

Or

(b) On every Sunday morning, a dental hospital renders free dental service to the patients. As per the hospital rules, 3 dentists who are equally qualified and experienced will be on duty then. It takes on an average 10 minutes for a patient to get treatment and the actual time taken is known to vary approximately exponentially around this average. The patients arrive according to the Poisson distribution with an average of 12 per hour. The hospital management wants to investigate the following:

(i) The expected number of patients waiting in the queue. (8)

(ii) The average time that a patient spends at the hospital (iii) The expected percentage of idle time for each dentist (iv) The fraction of time at least one dentist is idle. (8)

15. (a) Derive Pollaczek — Khintchine formula of an M/G/1 queue.

Or

(b) Write short notes on :

(i) open queueing networks and (8)

(ii) closed queueing networks. (8)

06/01/2024 - FN

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 70857

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours Maximum : 100 marks

(Use of Statistical table is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If X is a Poisson random variable with parameter $\lambda > 0$, then prove that $E(X^2) = \lambda E(X+1)$.

2. Test whether $f(x) = \begin{cases} |x|, & -1 \le x \le 1 \\ 0, & otherwise \end{cases}$ can be the probability density function of a continuous random variable.

3. Random variables X and Y have a joint probability density function $f(x, y) = \begin{cases} c\ ; & 0 \le x \le 5,\ 0 \le y \le 3 \\ 0\ ; & otherwise \end{cases}$. Find the constant '$c$'.

4. State Central limit theorem.

5. Give an example for each of the following stochastic processes:

(a) Continuous time, Discrete state space

(b) Continuous time, Continuous state space.

6. The number of queries arriving to a university database system in t seconds is a Poisson process with Parameter 0.3. What is the probability that in 2 seconds there will be 3 queries? What is the probability that there are no queries in 3 seconds?

7. What is the steady state condition for the M/M/c queueing model? Does the same steady state condition hold for a finite M/M/c/K queue? Why or why not?

8. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a M/M/1 queueing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour?

9. What do you mean by Non-Markovian queueing models?

10. State Jackson's theorem for an open network.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A bolt is manufactured by 3 machines A, B and C. A turns out twice as many items as B and machines B and C produce an equal number of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and 1 is chosen from this pile. What is the probability that it is defective? (8)

(ii) If $p(x) = \begin{cases} x e^{\frac{-x^2}{2}}; & x \geq 0 \\ 0; & x < 0 \end{cases}$ then

(1) show that $p(x)$ is a pdf

(2) find its distribution function. (8)

Or

(b) (i) Find the moment generating function of the exponential distribution. (8)

(ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modelled by an N(5,16) normal random variable X.

(1) What is the probability that a randomly selected parcel weighs between 2 and 9 pounds?

(2) What is the probability that a randomly selected parcel weighs more than 8 pounds? (8)

12. (a) Let the joint PMF of X and Y be given by $p(x, y) = \begin{cases} \frac{1}{25}(x^2 + y^2)\ ; \ if\ x = 1, 2;\ y = 0, 1, 2 \\ 0 \qquad ; \ otherwise \end{cases}$.

(i) Find the Marginal PMFs. (5)

(ii) Determine $P(X > Y)$, $P(X + Y \leq 2)$ and $P(X + Y) = 2$. (4)

(iii) Are X and Y independent? Why or why not? (3)

(iv) Find $p_{X/Y}(x/y)$ and $P(X = 2/Y = 1)$. (4)

Or

(b) (i) Find the coefficient of correlation between X and Y, using following data: (8)

X:	1	3	5	7	8	10
Y:	8	12	15	17	18	20

(ii) The lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1200 hrs and standard deviation 250 hrs. Find the probability, using the central limit theorem, that the average lifetime of 60 bulbs exceeds 1250 hrs. (8)

13. (a) (i) Suppose that customers arrive at a bank, according to a Poisson process with a mean rate of 4 per minute; find the probability that during a time interval of 3 minutes

(1) exactly 5 customers arrive and

(2) more than 5 customers arrive. (8)

(ii) A particle performs a random walk with absorbing barriers, say at 0 and 4. Whenever it is in any position are $(0 < r < 4)$, it moves to $r + 1$ with probability 'p' or to $r - 1$ with probability 'q', $p + q = 1$. But as soon as it reaches 0 or 4, it remains there itself. Define Markov chain, find its TPM and state transition diagram. (8)

Or

(b) (i) An observer at a lake notices that when fish are caught, only 1 out of 9 trout is caught after another trout, with no other fish in between, whereas 10 out of 11 non-trout are caught following non-trout, with no trout between. Assuming that all fish are equally likely to be caught, what fraction of fish in the lake is trout? (8)

(ii) Experience shows that whether or not a bidder is successful in a bid depends on the success and failures of his previous two bids. If his last two bids were successful, his next bid will be successful with probability 0.5. If only one of his last two bids was successful, the probability is 0.6 that the next bid will be successful. If none of the last two bids were successful, the probability is 0.7 that the next one will be successful. Define the Markov chain, find its TPMP, and also find P^2. (8)