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Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 70861

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Fifth Semester

Computer Science and Engineering

MA 8551 – ALGEBRA AND NUMBER THEORY

(Common to Computer and Communication Engineering/Information Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the inverse of 2 under the binary operation * defined in R by $a * b = a + b - 1$.

2. Define a subgroup with an example.

3. Find all the roots of $f(x) = x^2 - 4$ in $Z_5[x]$.

4. Give an example of a polynomial that is irreducible in $R[x]$.

5. Find the number of positive integer's ≤ 1745 and not divisible by 13.

6. State Euclid's theorem.

7. Determine whether the LDE $6x + 12y + 15z = 10$ is solvable.

8. What is the remainder when 2^{35} is divided by 7.

9. State Fermat's little theorem.

10. Compute $\phi(n)$ for $n = 5!$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let G be the set of all rigid motions of a equilateral triangle. Identify the elements of G. Show that it is a non-abelian group of order six.

(ii) Show that $\{Z_4, +_4, \bullet_4\}$ is a ring. (8 + 8)

Or

(b) (i) Prove that every subgroup of a cyclic group is cyclic.

(ii) Find $[25]^{-1}$ in Z_{72}. (8 + 8)

12. (a) Prove that a finite field F has order p^t, where p is a prime and $t \in Z^+$. (16)

Or

(b) (i) If $f(x) = 3x^5 - 8x^4 + x^3 - x^2 + 4x - 7$, $g(x) = x + 9$ and $f(x), g(x) \in \mathbf{Z}_{11}[x]$, find the remainder when $f(x)$ is divide by $g(x)$.

(ii) Find the gcd of $2x^3 + 2x^2 - x - 1$ and $2x^4 - x^2$ in $Q[x]$. (8 + 8)

13. (a) (i) Prove that $n^4 + 2n^3 + n^2$ is divisible by 4.

(ii) Apply Euclidean algorithm to express the gcd of 4076 and 1024 as a linear combination of themselves. (8 + 8)

Or

(b) (i) Prove that there are infinitely many primes of the form $4n + 3$.

(ii) Using recursion, evaluate $[24, 28, 36, 40]$. (8 + 8)

14. (a) (i) Find the general solution of the linear Diophantine equation $2x + 3y = 4$.

(ii) Find the incongruent solutions of $19x \equiv 29 \pmod{16}$. (8 + 8)

Or

(b) State and prove Chinese Remainder Theorem. Using it find the least positive integer that leaves the remainder 1 when divided by 3, 2 when divided by 5 and 3 when divided by 7. (16)

15. (a) (i) State and prove Wilson's Theorem.

(ii) Using Euler's theorem find the remainder when 7^{1020} is divided by 15. (8 + 8)

Or

(b) (i) If m and n are relatively prime then show that the Euler's Phi function satisfies $\phi(mn) = \phi(m)\phi(n)$ and hence compute the value of $\phi(1976)$.

(ii) Compute tau and sigma functions for $n = 6120$. (8 + 8)