Reg. No. :

Question Paper Code : 83219

M.E./M.Tech. DEGREE EXAMINATION, JANUARY 2014.

First Semester

Computer Science and Engineering

MA 7155 — APPLIED PROBABILITY AND STATISTICS

(Common to M.E. Software Engineering/M.E. Computer Science and Engineering (With Specialization in Networks) M. Tech. Information Technology/M.E. Biometrics Cyber Security/M.E. Multimedia Technology/M.E. Industrial Engineering)

(Regulation 2013)

Time : Three hours　　　　Maximum : 100 marks

(Statistical tables are permitted into the examination hall)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A continuous random variable X has a density function given by $f(x) = k(1+x),\ 2 \le x \le 5$. Find the value of k.

2. Check whether the following data follows a binomial distribution or not : mean = 3, variance = 4.

3. Define marginal probability function of the random variables X and Y.

4. The two lines of regression are $8x - 10y + 66 = 0,\ 40x - 18y - 214 = 0$. Find the mean values of X and Y.

5. Define the unbiasedness of an estimator.

6. Write the normal equations for fitting a straight line by the method of least squares.

7. Explain Null hypothesis and alternative hypothesis.

8. State any two applications of chi square distribution.

9. If $X = \begin{pmatrix} 42 & 4 \\ 52 & 5 \\ 48 & 4 \end{pmatrix}$, find $\overline{X}$.

10. State any two properties of multi variate normal distribution.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) The diameter of an electric cable, say X is assumed to be a continuous random variable with p.d.f $f(x) = 6x(1-x),\ 0 \le x \le 1$. Check that $f(x)$ is a p.d.f. and determine 'a' such that $P(X < a) = P(X > a)$. (8)

(ii) Let the random variable X assume the value r with the probability law : $P(X = r) = q^{r-1}p; r = 1,2,3.....$ Find the moment generating function of X and hence its mean. (8)

Or

(b) (i) If X is uniformly distributed over (0, 10) find the probability that $X < 2$; $X > 8$; $3 < X < 9$. (8)

(ii) If the p.d.f of a random variable X is $f(x) = 2x$, $0 < x < 1$, find the p.d.f of $Y = e^{-X}$. (8)

12. (a) (i) If the joint p.d.f of (X, Y) is given by $p(x,y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find the marginal distributions of X and Y. (8)

(ii) The joint density function of X and Y is $f(x,y) = \begin{cases} e^{-(x+y)}, & 0 \le x,\ \ y \le \infty \\ 0, & \text{otherwise} \end{cases}$. Are X and Y are independent. (8)

Or

(b) Two random variables X and Y have the following joint probability density function, $f(x,y) = \begin{cases} 2 - x - y, & 0 \le x \le 1;\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$. Find the correlation coefficient of X and Y. (16)

13. (a) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. Show that the sample mean $\overline{x}$ is sufficient for estimating the parameter λ of the Poisson distribution. (16)

Or

(b) Fit a straight line $y = a + bx$ for the following data by the principle of least squares.

x:	0	1	2	3	4
y:	1	1.8	3.3	4.5	6.3

Also find the value of y when $x = 1.5$ (16)

14. (a) (i) In a sample of 1000 people in Karnataka 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 5% level of significance? (8)

(ii) The following table gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week. (8)

Days :	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents :	14	18	12	11	15	14

Or

(b) (i) The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D. 2.5 inches? (8)

(ii) In an experiment on immunization of cattle from tuberculosis the following results were obtained. (8)

	Affected	Not affected
Inoculated	12	26
Not inoculated	16	6

Calculate the chi square and discuss the effect of vaccine in controlling susceptibility to tuberculosis.

15. (a) Consider the random vector $X' = \{X_1, X_2\}$. The discrete random variable X_1 have the following probability function : (16)

x_1 :	-1	0	1
$P_1(x_1)$:	0.3	0.3	0.4

and X_2 have the following probability function :

x_2 :	0	1
$P_2(X_2)$:	0.8	0.2

Find the covariance matrix for the two random variables X_1 and X_2 when their joint p.d.f $p_{12}(x_1, x_2)$ is given by

x_1 \ x_2	0	1
–1	0.24	0.06
0	0.16	0.14
1	0.40	0.00

Or

(b) For the covariance matrix $\Sigma = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix}$, the derived correlation matrix $P = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$, show that the principal components obtained from covariance and correlation matrices are different. (16)
