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Reg. No. | | | | | | | | | | | | |

Question Paper Code : 66191

M.E./M.Tech. DEGREE EXAMINATION, DECEMBER 2015 / JANUARY 2016

First Semester

Computer Science and Engineering

MA7155 : APPLIED PROBABILITY AND STATISTICS

(Common to M.E. Biometrics and Cyber Security M.E. Computer Science and Engineering with specialization in Networks) M.E. Industrial Engineering M.E. Multimedia Technology M.E. Software Engineering and M.Tech. Information Technology)

(Regulations – 2013)

(Statistical tables is to be provided)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. If the probability mass function of a random variable X is given by $P(X = r) = kr^3$, where $r = 1,2,3,4$. Find the value of k and distribution function of X.

2. If the distribution function of a random variable X is given by

$$F(x) = \begin{cases} 1 - \dfrac{4}{x^2}, & \text{for } x > 2 \\ 0, & \text{for } x \le 2 \end{cases}, \text{ find } P(5 < X < 6)$$

3. If the joint pdf of X and Y is given by $f(x, y) = \frac{8}{9}xy$, $1 < x < y < 3$, find the marginal density function of X.

4. If the joint pmf of X and Y is given by

$$P(x, y) = \begin{cases} C(x + y), & x = 0, 1, 2 \text{ and } y = 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}, \text{ find C and } P(X = 1).$$

5. Give the normal equations to fit the parabola $y = a + bx + cx^2$.

6. Can $Y = 5 + 2.8x$ and $X = 3 - 0.5\,y$ be the estimated regression equations of y on x and x on y respectively ? Explain.

7. Define type I and type II errors.

8. Give the value of χ^2 for a 2×2 contingency table with cell frequencies a, b, c and d.

9. Define second principle component.

10. If X_1 and X_2 are two uncorrelated random variables, then what is the correlation coefficient matrix ?

PART – B (5 × 16 = 80 Marks)

11. (a) (i) An insurance company found that only 0.005% of the population is involved in a certain type of accident each year. If its 2000 policyholders were randomly selected from the population, what is the probability that not more than two of its clients are involved in such an accident next year ? **(8)**

(ii) The daily consumption of milk in a city in excess of 20,000 gallons is approximately distributed as a Gamma variate with parameters $k = 2$ and $\lambda = \frac{1}{10{,}000}$. The city has a daily stock of 30,000 gallons. What is the probability that the stock is insufficient on a particular day ? **(8)**

OR

(b) (i) Find the moment generating function of Binomial distribution and hence obtain its mean and variance. **(8)**

(ii) In an intelligence test administered on 1000 children the average score is 42 and standard deviation 24. Assuming the normal distribution,

(1) Find the number of children exceeding the score 50 and

(2) Find the number of children with score lying between 30 and 54. **(8)**

12. (a) (i) In a partially destroyed laboratory data, only the equations giving the two lines of regression of y on x and x on y are available and are respectively $7x - 16y + 9 = 0$, $5y - 4x - 3 = 0$. Calculate the coefficient of correlation. **(8)**

(ii) The joint probability density function of a random variable (X, Y) is given by $f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \le x \le 1, 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$. Find $P(X + Y \ge 1)$ and $P\left(Y < \frac{1}{2} / X < \frac{1}{2}\right)$. **(8)**

OR

(b) (i) If the probability of a two discrete random variables X and Y is given by

$$f(x, y) = \begin{cases} k(x + 2y); & x = 0, 1, 2 \text{ and } y = 0, 1, 2 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

(1) Find k, (2) Find the marginal distributions and conditional distribution of Y for $X = x$. **(8)**

(ii) The joint probability density function of a two dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} 4xye^{-(x^2 + y^2)} \ ; & x \geq 0, y \geq 0 \\ 0 \quad , & \text{otherwise} \end{cases}$$, find the density function of $U = \sqrt{X^2 + Y^2}$. **(8)**

13. (a) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a population with unknown mean μ. Consider the following estimators to estimate μ.

$$t_1 = \frac{(x_1 + x_2 + x_3 + x_4 + x_5)}{5}, t_2 = \frac{(x_1 + x_2)}{2} + X_3 \text{ and } t_3 = \frac{(2x_1 + x_2 + \lambda x_3)}{3}$$ where λ is such that t_3 is an unbiased estimator of μ. Find λ. Are t_1 and t_2 unbiased ? State giving reasons, the estimator which is best among t_1, t_2 and t_3. **(16)**

OR

(b) (i) Fit a straight line $y = a + bx$ to the following data, using principle of least square : **(8)**

x **:**	1	2	3	4	6	8
y :	2.4	3	3.6	4	5	6

(ii) Find the most likely price in Bombay corresponding to the price of ₹ 70 at Calcutta from the following :

	Calcutta	**Bombay**
Average Price	65	67
Standard Deviation	2.5	3.5

Correlation coefficient between the prices of commodities in the two cities is 0.8. **(8)**

14. (a) (i) Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is a significant decrease in the consumption of tea after the increase in duty. **(8)**

(ii) Two researchers A and B adopted different techniques while rating the students level. Can you say that the techniques adopted by them are significant at 5% level ? **(8)**

Researchers	**Below average**	**Average**	**Above average**	**Genius**	**Total**
A	40	33	25	2	100
B	86	60	44	10	200
Total	126	93	69	12	300

OR

(b) (i) Samples of two types of electric bulbs were tested for length of life and the following data were obtained.

	Size	Mean	S.D.
Sample I	8	1234 hours	36 hours
Sample II	7	1036 hours	40 hours

Is the difference in means sufficient to warrant that type I bulbs are superior to type II bulbs at 5% level ? **(8)**

(ii) From the following two sample values, find out whether they have come from the same population at 5% level. **(8)**

Sample I :	17	27	18	25	27	29	27	23	17
Sample II :	16	16	20	16	20	17	15	21	

15. (a) Find the mean matrix, covariance matrix, standard deviation matrix and correlation coefficient matrix for two random variables X_1 and X_2 whose joint mass function is given by. **(16)**

x_1/x_2	0	1
−1	0.24	0.06
0	0.16	0.14
1	0.40	0.0

OR

(b) (i) Calculate the population principal component of the random variables X_1, X_2, X_3 whose covariance matrix is $\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. **(8)**

(ii) Let X be distributed as $N_3(\mu, \Sigma)$ where $\mu' = (1, -1, 2)$ and $\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$, which of the following random variables are independent ? Explain.

(1) X_1 and X_2

(2) X_1 and X_3

(3) X_2 and X_3

(4) (X_1, X_3) and X_2 **(8)**
