Reg. No. :

Question Paper Code : 13649

M.E./M.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

First Semester

Computer Science and Engineering

MA 7155 — APPLIED PROBABILITY AND STATISTICS

(Common to M.E. Industrial Engineering, M.E. Software Engineering, M.E. Computer Science and Engineering (with specialization in Networks) M.E. Biometrics and Cyber Security, M.E. Multimedia Technology and M.Tech. Information Technology)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. It has been claimed that in 60% of all solar – heat installations the utility bill is reduced by at least one-third. Accordingly, what are the probabilities that the utility bill will be reduced by at least one-third in four of five installations?

2. Find k so that the following can serve as the probability density of a random variable : $f(x)=\begin{cases} 0 & for\ \ x \le 0 \\ k\,xe^{-4x^2} & for\ \ x>0 \end{cases}$.

3. Define the probability mass function of two-dimensional random variables (X, Y).

4. Let $f(x, y)=k$ when $8 \le x \le 12$ and $0 \le y \le 2$ and zero elsewhere. Find k.

5. An industrial engineer intends to use the mean of a random sample of size $n = 150$ to estimate the average mechanical aptitude (as measured by a certain test) of assembly line workers in a large industry. If, on the basis of experience, the engineer can assume that $\sigma = 6.2$ for such data, what can he assert with probability 0.99 about the maximum size of his error?

6. What is meant by maximum likelihood estimator?

7. Define : Type I and Type II errors with exmaples.

8. State the null and alternate hypothesis in each case.

(a) A hypothesis test will be used to potentially provide evidence that the population mean is greater than 10.

(b) A hypothesis test will be used to potentially provide evidence that the population mean is not equal to 7.

9. Define : Random vectors and Random matrices.

10. State any two properties of the Multivariate Normal distribution.

PART B — ($5 \times 16 = 80$ marks)

11. (a) Find the Probabilities that a random variable having the standard normal distribution will take on a value

(i) between 0.87 and 1.28

(ii) between –0.34 and 0.62

(iii) greater than 0.85

(iv) greater than –0.65.

Or

(b) Find the Moment generating function for Poisson distribution and Gamma distribution.

12. (a) Show that the following function satisfies the properties of a joint probability mass function

X	1	1.5	1.5	2.5	3
Y	1	2	3	4	5
$f_{XY}(x, y)$	1/4	1/8	1/4	1/4	1/8

Determine the following :

(i) $P(X < 2.5, Y < 3)$

(ii) $P(X < 2.5)$

(iii) $P(Y < 3)$

(iv) $P(X > 1.8, Y = 4.7)$

(v) $E(X), E(Y), Var(X)$ and $Var(Y)$,

(vi) Marginal probability distribution of the random variable X

(vii) Conditional probability distribution of Y given that X = 1.5.

Or

(b) Let the random variable X denote the time until a computer server connects to your machine (in miliseconds), and let Y denote the time until the server authorizes you as a valid user (in miliseconds). Each of these random variables measures the wait from a common starting time and $X < Y$. Can we assume the following function as a joint probability density function for X and Y?

$$f_{XY}(x, y) = 6 \times 10^{-6} \exp(-0.001x - 0.002y)$$
$$for \quad x < y$$

Determine the probability that X<1000 and Y<2000.

13. (a) (i) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample of size n from the Poisson distribution $f(x/\lambda) = \dfrac{\lambda^x e^{-\lambda}}{x!}$ where $0 \le \lambda < \infty$. Obtain the maximum likelihood estimator of λ.

(ii) Let $X_1, \ldots\ldots, X_n$ be a random sample of size 'n' from a normal distribution with known variance. Obtain the maximum likelihood estimator of μ.

Or

(b) The following are measurements of the air velocity and evaporation co-efficient of burning fuel droplets in an impulse engine :

Air velocity (cm/s) x	20	60	100	140	180	220	260	300	340	380
Evaporation coefficient (mm²/s) y	0.18	0/37	0.35	0.78	0.56	0.75	1.18	1.36	1.17	1.65

Fit a straight line to these data by the method of least squares, and use it to estimate the evaporation co-efficient of a droplet when the air velocity is 190 cm/s.

14. (a) (i) The specifications for a certain kind of ribbon call for a mean breaking strength of 180 pounds. If five pieces of the ribbon (randomly selected from different rolls) have a mean breaking strength of 169.5 pounds with standard deviation of 5.7 pounds, test the null hypothesis that the mean breaking strength $\mu = 180$ pounds against the alternative hypothesis $\mu < 180$ pounds at the 0.01 level of significance. Assume that the population distribution is normal.

(ii) It is desired to determine whether there is less variability in the silver plating done by Company 1 than that done by Company 2. If indepent random samples of size 12 of the two companies work yield $s_1 = 0.035$ mil and $s_2 = 0.062$ mil, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $\sigma_1^2 < \sigma_2^2$ at the 0.05 level of significance.

Or

(b) (i) To determine whether there really is a relationship between an employee's performance in the company's training program and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtains the results shown in the following table ;

		Performance in training program			
		Below average	Average	Above average	Total
Success in job (employer's rating)	Poor	23	60	29	112
	Average	28	79	60	167
	Very good	9	49	63	121
	Total	60	188	152	400

Use the 0.01 level of significance to test the null hypothesis that performance in the training program and success in the job are independent.

(ii) The Lapping process which is used to grind certain silicon wafers to the proper thickness is acceptable only if σ, the population standard deviation of the thickness of dice cut from the wafers, is at most 0.50 mil. Use the 0.05 level of significance to test the null hypothesis $\sigma = 0.50$ against the alternative hypothesis $\sigma > 0.50$, if the thickness of 15 dice cut from such wafers have a standard deviation of 0.64 mil.

15. (a) Suppose the random variables X_1, X_2 and X_3 have the covariance matrix $\sum = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Calculate the population principal components and hence find variances and covariances of principal components.

Or

(b) (i) If X is distributed as $N_5(\mu, \varepsilon)$, find the distribution of $\begin{bmatrix} X_2 \\ X_4 \end{bmatrix}$.

(ii) Let $\underset{(3\times1)}{X}$ be $N_3(\mu, \varepsilon)$ with $\sum = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Are X_1 and X_2 independent? What about (X_1, X_2) and X_3?

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