Reg. No. :

Question Paper Code : 71438

M.E./M.Tech. DEGREE EXAMINATION, JUNE/JULY 2013.

First Semester

Computer Science and Engineering

MA 9219/MA 9329/MA 904/UMA 9110/UMA 9128 — OPERATIONS RESEARCH

(Common to M.E. Software Engineering, M.E. Network Engineering and M.E. Computer Networks, M.E. Computer Networking and Engineering and M.Tech Information Technology).

(Regulation 2009/2010)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the laws of motion for Birth-Death processes.

2. Define Little's queuing formula.

3. Write the Kendall's notation for queuing system.

4. What are the assumptions of Erlang's queuing theory to control the queuing system?

5. Write the steps involved in inverse transformation method.

6. What are the stages exist in simulation process?

7. State the Primal and Dual relationship.

8. Apply graphical method to solve the L.P.P.

   Maximise $Z = 4x_1 + 3x_2$

   subject to $4x_1 + 2x_2 \le 10$

   $2x_1 + \frac{8}{3}x_2 \le 8$,

   $x_2 \le 6$ and $x_1, x_2 \ge 0$

9. If a company charges a price P for a product, then it can sell $3e^{-p}$ thousand units of product. Then, f(p)= 3,000pe^{-p} is the company's revenue if it charges a price p. Suppose the current price is \$4 and the company increases the price by 5¢. By approximately how much would the company's revenue change? Given $\Delta p = 0.05$.

10. List out the cases for calculating the extremum candidates.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Derive the steady-state probabilities for Birth-Death Processes. (8)

(ii) My home uses two light bulbs. On average, a light bulb lasts for 22 days (exponentially distributed). When a light bulb burns out, it takes an advantage of 2 days (exponentially distributed) before I replace the bulb.

(1) Formulate a three-state-birth-death model of this situation.

(2) Determine the fraction of the time that both light bulbs are working.

(3) Determine the fraction of the time that no light bulbs are working. (8)

Or

(b) (i) The manager of a bank must determine how many tellers should work on Fridays. For every minute a customer stands in line, the manager believes that a delay cost of 5¢ is incurred. An average of 2 customers per minute arrives at bank. On the average, it takes a teller 2 minutes to complete a customer's transaction It cost the bank \$9 per hour to hire a teller. Interarrival times and service times are exponential. To minimize the sum of service costs and delay costs, how many tellers should the bank have working on Fridays? (8)

(ii) A service facility consists of one server who can serve an average of 2 customers per hour (service times are exponential). An average of 3 customers per hour arrives at the facility (interarrival times are assumed exponential) The system capacity is 3 customers.

(1) On the average, how many potential customers enter the system each hour?

(2) What is the probability that the server will be busy? (8)

12. (a) (i) Derive L, Lq, W and Wq for machine repair models. (8)

(ii) The Gotham Township Police Department has 5 patrol cars. A patrol car breaks down and requires service once every 30 days. The police department has two repair workers, each of whom takes an average of 3 days to repair a car. Breakdown times and repair times are exponential.

(1) Determine the average number of police cars in good condition.

(2) Find the average down time for a police car that needs repairs.

(3) Find the fraction of the time a particular repair worker is idle. (8)

Or

(b) Consider two servers. An average of 8 customers per hour arrive from outside at server 1, and an average of 17 customers per hour arrive from outside at server 2. Interarrival times are exponential. Server I can serve at an exponential rate of 20 customers per hour and server 2 can serve at an exponential rate of 30 customers per hour. After completing service at server I, half of the customers leave the system, and half go to server 2, 3/4 of the customer's complete service and ¼ returns to server 1.

(i) What fraction of the time is the server 1 idle?

(ii) Find the expected number of customers at each server.

(iii) Find the average time a customer spends in the system.

(iv) How would the answers to parts 1 – 3 change if server 2 could serve only an average of 20 customers per hour? (16)

13. (a) Explain acceptance-rejection method to generate random variates for continuous distributions and solve the following:

Generate random variates from a triangular distribution whose pdf is given by

$$f(x) = \begin{cases} -1/6 + x/12, \text{if } 2 \le x \le 6 \\ 4/3 - x/6, \text{ if } 6 \le x \le 8 \end{cases}$$ (16)

Or

(b) The pdf of exponential distribution is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, \text{ if } x \ge 0, \lambda > 0 \\ 0, \text{ otherwise} \end{cases}$$ (16)

Use the inverse transformation method to generate observations from an exponential distribution.

14. (a) Solve using the Simplex Method the following problem:

Maximize $Z = 3x + 2y$

subject to: $2x + y \leq 18$

$2x + 3y \leq 42$

$3x + y \leq 24$ and $\left.\begin{matrix} x \geq 0, \\ y \geq 0 \end{matrix}\right\}$ (16)

Or

(b) Solve the transportation problem to find the optimal solution (16)

	To				Supply
	8	6	10	9	35
From	9	12	13	7	50
	14	9	16	5	40
Demand	45	20	30	30	

15. (a) (i) A company is planning to spend \$10,000 on advertising. It costs \$3,000 per minute to advertise on television and \$1,000 per minute to advertise on radio. If the firm buys x minutes of television advertising and y minutes of radio advertising then its revenue in thousands of dollars is given by $f(x, y) = -2x^2 - y^2 + xy + 8x + 3y$. How can the firm maximize its revenue? (8)

(ii) Use the K-T conditions to find the optimal solution to the following NLP:

Min $Z = (x_1 - 1)^2 + (x_2 - 2)^2$

S.T. $-x_1 + x_2 = 1$

$x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$ (8)

Or

(b) Solve the following Quadratic Programming Problem using Wolfe's Method.

Min $z = -x_1 - x_2 + \left(\frac{1}{2}\right)x_1^2 + x_2^2 - x_1x_2$

Subject to $x_1 + x_2 \leq 3$ (16)

$-2x_1 - 3x_2 \leq -6$

and $x_1, x_2 \geq 0$