ii) The following data are collected on two characters : **(7)**

	Smokers	Non-Smokers
Literates	83	57
Illiterates	45	68

Based on this, can you say that there is no relation between smoking and literacy ?

15\. a) Find the covariance matrix for the two random variables X_1 and X_2 when their joint probability function $p_{12}(x_1, x_2)$ is represented by the entries in body of the following table :

$x_1 \backslash x_2$	0	1	$p_1(x_1)$
– 1	0.24	0.06	0.3
0	0.16	0.14	0.3
1	0.40	0.00	0.4
$p_2(x_2)$	0.8	0.2	

(OR)

b) For the covariance matrix $\Sigma = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix}$, the derived correlation matrix $\rho = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$, show that the principal components obtained from the covariance and correlation matrices are different.

PART – C **(1×15=15 Marks)**

16\. a) i) If a random variable X has the pdf $f(x) = \begin{cases} \frac{1}{4}, & |x| < 2 \\ 0, & \text{otherwise} \end{cases}$

Find $P(X < 1)$, $P(|x| > 1)$, $P(2X + 3 > 5)$. **(7)**

ii) If X is uniformly distributed over (0, 10) find the probability that $X < 2$, $X > 8$, $3 < X < 9$. **(8)**

(OR)

b) Let X and Y be random variables having joint density function

$$f(x, y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \le x, y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

. Find the correlation coefficient r_{XY}.

Reg. No. :

Question Paper Code : 47233

M.E./M.Tech. DEGREE EXAMINATION, JANUARY 2018

First Semester

Biometrics and Cyber Security

MA 5160 – APPLIED PROBABILITY AND STATISTICS

(Common to M.E. Computer Science and Engineering/M.E. Computer Science and Engineering (With Specialization in Networks)/M.E. Industrial Engineering/ M.E. Manufacturing Engineering/M.E. Multimedia Technology/M.E. Software Engineering/M.Tech. Information Technology)

(Regulations : 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

(Statistical Table may be permitted)

PART – A **(10×2=20 Marks)**

1\. Given $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{4}$. State whether the events A and B are

i) mutually exclusive

ii) independent.

2\. The mean and standard deviation of a Binomial distribution are respectively 4 and $\sqrt{\frac{8}{3}}$. Find the values of q and p.

3\. Define joint pdf for continuous random variables X and Y.

4\. X and Y are random variables with joint pdf $f(x,y) = \begin{cases} x + y, & 0 < x, y < 1 \\ 0, & \text{otherwise} \end{cases}$ check whether X and Y are independent.

5\. Define likelihood function.

6\. The two regression lines are $4x - 5y + 33 = 0$, $20x - 9y = 107$. Find the means of X and Y.

7\. Explain the term null hypothesis.

8. Give two uses of Chi-square distribution.

9. Define random vector and random matrix.

10. Suppose the covariance matrix $\sum = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{pmatrix}$, obtain $V^{\frac{1}{2}}$ and ρ.

PART – B (5×13=65 Marks)

11. a) i) The contents of urns I, II and III are as follows :
1 white, 2 black and 3 red balls ;
2 white, 1 black and 1 red balls ;
4 white, 5 black and 3 red balls.
One urn is chosen at random and two balls drawn from it. They happen to be white and red. What is the probability that they come from urns I, II or III ? (6)

ii) A random variable has the pdf given by $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$. Find the moment generating function, the first four moments about the origin. (7)

(OR)

b) i) If X is a Geometric variable taking values 1, 2, 3, ... ∞, find P(X is odd). (6)

ii) A manufacturer produce air mail envelops whose weight is normal with mean $\mu = 1.950$ gm and S.D. $\sigma = 0.025$gm. The envelops are sold in lots of 1000. How many envelops in a lot may be heavier than 2 grams ? (7)

12. a) The two dimensional random variable (X, Y) has the joint probability mass function $f(x,y) = \frac{x+2y}{27}$, x = 0, 1, 2, ; y = 0, 1, 2. Find the conditional distribution of Y for X = x.

(OR)

b) The joint density function of X and Y is $f(x,y) = \begin{cases} e^{-(x+y)}, & 0 \leq x, y \leq \infty \\ 0, & \text{otherwise} \end{cases}$

i) Are X and Y independent ?
Find :
ii) P(X < 1)
iii) P(X + Y < 1). (6+3+4)

13. a) i) Let $x_1, x_2, ... x_n$ be a random sample from the uniform distribution with pdf $f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 < x < \infty, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$ obtain the maximum likelihood estimator for θ. (6)

ii) Find the best fit values of a and b so that y = a + bx fits the data given in the table : (7)

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

(OR)

b) i) Estimate α and β in the case of Pearson's Type III distribution by the method of moments : $f(x;\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, 0 \leq x \leq \infty$. (6)

ii) Obtain the equations of the lines of regression from the following data : (7)

X :	1	2	3	4	5	6	7
Y :	9	8	10	12	11	13	14

14. a) i) Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is significant decrease in the consumption of tea after the increase in duty. (6)

ii) Two independent samples of eight and seven items respectively had the following values of the variable :

Sample 1 :	9	11	13	11	15	9	12	14
Sample 2 :	10	12	10	14	9	8	10	

Do the two estimates of population variance differ significantly at 5% level of significance ? (7)

(OR)

b) i) A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D. of 6.4 cm. while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that Americans are on the average taller than the English men ? (6)