(b) (i) The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the avenge height is greater than 64 inches? Test at 5 % significance level assuming that for 9 degrees of freedom $P(t > 1.83) = 0.05$. (7)

(ii) The table below gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week. (6)

Days	Mon	Tues	Wed	Thur	Fri	Sat
No. of accidents :	14	18	12	11	15	14

15. (a) Calculating the population principal components of random variables X_1, X_2, X_3 have the covariance matrix $\begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and also find $Var(Y_1)$, $Cov(Y_1, Y_2)$ and total population variance. (13)

Or

(b) Computing the correlation matrix from the Covariance matrix $\begin{pmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{pmatrix}$. (13)

PART C — (1 × 15 = 15 marks)

16. (a) The joint probability density function of the two dimensional random variable (X, Y) is $f(x,y) = \begin{cases} 2-x-y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$, Find the correlation coefficient between X and Y. (15)

Or

(b) (i) In a city the daily consumption of electric power is million kilowatts hours is a random variable with general Gamma distribution with parameter $\lambda = 1/2$ and $k = 3$. If the power plant of this city has a daily capacity of 12 million kilowatts hours, what is the probability that this power supply will be inadequate on any given day? (8)

(ii) Two batches each of 12 animals are taken for test of inoculation. One batch was inoculated and the other batch was not inoculated. The frequencies of dead and surviving animals are given below in both cases. Can the inoculation be regarded as effective against the disease. (7)

	Dead	Survived
Inoculated	2	10
Not inoculated	8	4

Reg. No. : [ ]

Question Paper Code : 40731

M.E./M.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

First Semester

Biometrics and Cyber Security

MA 5160 — APPLIED PROBABILITY AND STATISTICS

(Common to Computer Science and Engineering/Computer Science and Engineering (With Specialization in Networks)/Industrial Engineering/ Manufacturing Engineering/Multimedia Technology/Software Engineering and Information Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Statistical tables are permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A family has two children. What is the probability that both are boys given that at least one of them is a boy?

2. A random variable X has the probability function $f(x) = \frac{1}{2^x}$, $x = 1, 2, 3, \ldots$ Find its Moment generating function.

3. The joint probability density function of the random variable (X, Y) is given by $f(x,y) = 4xy\, e^{-(x^2+y^2)}, x > 0,\ y > 0$. Prove that X and Y are independent.

4. If X has mean 4 and variance 9, while Y has mean –2 and variance 5 and the two variables are independent, find $E(XY^2)$.

5. Define an Unbiased estimator.

6. What is meant by the least squares method?

7. Write any two applications of chi-square test.

8. A random sample of 500 robots was taken from an automobile consignment and 65 were found to be improperly built robots. Find the percentage of the improperly built robots in the consignment.

9. Compute expected values for discrete random vector $X' = [X_1, X_2]$ where $E(X_1) = 0.1$ and $E(X_2) = 0.2$.

10. Define first principle component.

PART B — (5 × 13 = 65 marks)

11. (a) (i) A continuous random variable X has probability density functions $f(x) = k x^2 e^{-x}$, $x \geq 0$. Find k, r^{th} moment, mean and variance. (7)

(ii) State and prove the memory less property of an exponential distribution. (6)

Or

(b) (i) If, on an average, 9 ships out of 10 arrive safely to a port, obtain the mean and standard deviation of the number of ships returning safely out of 150 ships. (7)

(ii) Find moment generation function of a Normally distributed random variable. (6)

12. (a) (i) If X and Y are two random variables having a probability density function

$$f(x,y) = \begin{cases} \frac{(6-x-y)}{8}; & 0 < x < 2,\ 2 < y < 4 \\ 0; & \text{otherwise} \end{cases}$$

Find

(1) $P(X < 1 \cap Y < 3)$

(2) $P(X < 1/Y < 3)$

(3) $P(X + Y < 3)$ (7)

(ii) The joint probability function of (X, Y) is given by $P(x, y) = K(2x + 3y)$, $x = 0, 1, 2;$ $y = 1, 2, 3$. Find the marginal distributions and conditional distributions. (6)

Or

(b) Marks obtained by 10 students in Mathematics X and statistics Y are given below :

X:	60	34	40	50	45	40	22	43	42	64
Y:	75	32	33	40	45	33	12	30	34	51

Find the two regression lines. Also find Y when $X = 55$. (13)

13. (a) (i) Fit the exponential curve $y = a e^{bx}$ to the following data, e being Napierian base, 2.71828 : (7)

x:	0	2	4
y:	5.012	10	31.62

(ii) A random sample $x_1, x_2, \ldots, x_n$ is drawn from the exponential population with density function $f(x; \alpha, \beta) = y_0\, e^{-\beta(x-\alpha)}$, $\alpha \leq x < \infty$, $\beta > 0$, y_0 being a constant, obtain the maximum likelihood estimators for α and β. (6)

Or

(b) State and Prove Cramer-Rao inequality. Given the p.d.f $f(x_i, \theta) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)^2}$ $-\infty < x < \infty$, $-\infty < \theta < \infty$; Show that the Cramer-Rao bound of variance of an unbiased estimators of θ is $2/n$, where n is the size of the random sample from this distribution. (13)

14. (a) (i) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same, at 5% level. (7)

(ii) It is known that the diameters of rivets produced by two firms A and B are practically the same but the standard deviation may differ. For 22 rivets produced by firm A, the standard deviation is 2.9 mm, white for 16 rivets manufactured by firm B, the standard deviation is 3.8 mm. Compute the statistic you would use to test whether the products of firm A have the same variability as those of firm B and test its significance. (6)

Or