15. (a) (i) Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2$ in the interval $0 < x < 2\pi$.

(ii) Obtain a half range Fourier sine series for $f(x) = \begin{cases} \omega x, & 0 \le x \le \frac{1}{2} \\ \omega(l - x), & \frac{1}{2} \le x \le 1 \end{cases}$, and find the value of the series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \infty$. (8 + 8)

Or

(b) An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintened at temperature u_0 at all points and the other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state. (16)

Reg. No. : [ ]

Question Paper Code : 25140

B.E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third Semester

Electronics and Communication Engineering

MA 8352 — LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Electronics and Telecommunication Engineering/ Medical Electronics/Biomedical Engineering/Computer and Communication Engineering)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Determine whether the vectors $v_1 = (1, -2, 3)$, $v_2 = (5, 6, -1)$, $v_3 = (3, 2, 1)$ form a linearly dependent or linearly independent set in R^3.

2. What are the possible subspace of R^2?

3. Verify that $T : R^3 \to R$, and $T(u) = \|u\|$ is a linear transformation or not.

4. State the dimension theorem for matrices.

5. Let R^2 have the weighted Euclidean inner product defined as $< u, v >= 2u_1 v_1 + 3u_2 v_2$ and let $u = (1, 1), v = (3, 2), w = (0, -1)$. Compute the value of $< u + v, 3w >$.

6. Let P_2 have the inner product $< p, q >= \int_{-1}^{1} p(x)\, q(x)\, dx$. Find the angle between p and q, where $p = x$ and $q = x^2$ with respect to the inner product on P_2.

7. How the second order partial differential equations are classified?

8. Solve $pq + p + q = 0$.

9. Find the value of c, for which $u = e^{-4t}\cos 3x$ is the solution of the equation $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.

10. State giving reasons whether the function $f(x) = \tan x$ can be expanded in Fourier series in the interval of $(-\pi, \pi)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let V be the set of all positive real numbers. Define the vector addition and scalar multiplication as follows :

$x + y = xy$ and $kx = x^k$.

Determine whether or not V is a vector space over $\mathbb{R}$ with respect to above operations.

(ii) Determine the basis and dimension of the solution space of the linear homogeneous system $x + y - z = 0; -2x - y + 2z = 0; -x + z = 0$. (8 + 8)

Or

(b) (i) Determine whether the set of all 2 × 2 matrix of the form $\begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix}$, $a, b \in R$, with respect to standard matrix addition and scalar multiplication is a vector space or not? If not, list all the axioms that fail to hold.

(ii) Determine whether the set of vectors $X_1 = (1, 1, 2)$, $X_2 = (1, 0, 1)$ and $X_3 = (2, 1, 3)$ span R^3. (10 + 6)

12. (a) (i) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x, y) = (x + 3y, 0, 2x - 4y)$. Compute the matrix of the transformation with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$. Find $N(T)$ and $R(T)$. Is T one-to-one? Is T onto. Justify your answer. (8)

(ii) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (2x - y, 3z)$ verify whether T is linear or not. Find $N(T)$ and $R(T)$ and hence verify the dimension theorem. (8)

Or

(b) (i) Let $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ be defined as

$T(f(x)) = f(x) + (x+1) f'(x)$

Find eigen values and corresponding eigen vectors of T with respect to standard basis of $\mathbb{P}_2(\mathbb{R})$. (8)

(ii) Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3, where $v_1 = (1, 1, 1)$, $v_2 = (1, 1, 0)$ and $v_3 = (1, 0, 0)$. Let $T : R^3 \to R^2$ be the linear transformation such that $T(v_1) = (1, 0)$, $T(v_2) = (2, -1)$ and $T(v_3) = (4, 3)$. Find the formula for $T(x_1, x_2, x_3)$, then use this formula to compute $T(2, -3, 5)$. (8)

13. (a) (i) State the projection theorem.

(ii) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis, where $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (0, 0, 1)$. (4 + 12)

Or

(b) Let the vector space P_2 have the inner product $< p, q > = \int_0^1 p(x)\, q(x)\, dx$.

Apply the Gram-Schmidt process to transform the basis $S = \{u_1, u_2, u_3\} = \{1, x, x^2\}$ into an orthonormal basis. (16)

14. (a) (i) Solve : $x(y - z)p + y(z - x)q = z(x - y)$.

(ii) Solve : $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$. (8 + 8)

Or

(b) (i) Form a partial differential equations by eliminating the function f from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

(ii) Solve : $(D^2 + DD' - 6D'^2)z = y\cos x$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. (6 + 10)