Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 40785

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Third Semester

Biomedical Engineering

MA 8352 — LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Computer and Communication Engineering/Electronics and Communication Engineering/Electronics and Telecommunication Engineering/ Medical Electronics)

(Regulations 2017)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Let V_1 and V_2 be subspaces of a vector space V. Is $V_1 \cup V_2$ a subspace of V?

2. Define linear independence of vectors.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(a_1, a_2) = (2a_1 + a_2, a_1)$. Is T linear?

4. Define range of a linear transformation.

5. If $\langle x, y \rangle$ is an inner product on a vector space V, then $2\langle x, y \rangle$ can be an inner product on V?

6. Define norm.

7. Define singular solution of a partial differential equation.

8. Solve $\frac{y^2 z}{x} p + xzq = y^2$.

9. State Dirichlet's conditions.

10. Write an example of two-dimensional heat equation.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Prove that any intersection of subspaces of a vector space V is a subspace of V. (8)

(ii) Prove that the span of any subset S of a vector space V is a subspace of V. (8)

Or

(b) (i) Let V be a vector space and let $S_1 \subseteq S_2 \subseteq V$. If S_2 is linearly independent, then prove that S_1 is linearly independent. (8)

(ii) Let V be a vector space with dimension n. Prove that any linearly independent subset of V that contains exactly n vectors is a basis for V. (8)

12. (a) State and prove 'Dimension theorem'. (16)

Or

(b) (i) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. (8)

(ii) Reduce the matrix $A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$ to the diagonal form. (8)

13. (a) (i) Explain the Gram-Schmidt process. (8)

(ii) In $\mathbb{R}^4$ the set $\{(1, 0, 1, 0), (1, 1, 1, 1), (0, 1, 2, 1)\}$ is linear independent. Using Gram-Schmidt process, obtain an orthonormal set. (8)

Or

(b) (i) Let V be a finite dimensional inner product space over F, and $g: V \to F$ be a linear transformation. Then prove that there exists a unique vector $y \in V$ such that $g(x) = \langle x, y \rangle$ for all $x \in V$. (8)

(ii) Let $A = M_{m \times n}(F)$. Then prove that rank $(A^* A) = \text{rank}(A)$. (8)

14. (a) (i) Solve $x^2 p^2 + y^2 q^2 = z^2$ (4)

(ii) Solve $\dfrac{\partial^3 z}{\partial x^3} - 2\dfrac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. (12)

Or

(b) (i) Solve $\left(D^2+2DD'+D'^2-2D-2D'\right)z=\sin\left(x+2y\right)$. (12)

(ii) Classify the partial differential equation

$$x^2\frac{\partial^2 z}{\partial x^2}+\left(1-y^2\right)\frac{\partial^2 z}{\partial y^2}=0,\ -\infty<x<\infty,\ -1<y<1\,. \quad (4)$$

15. (a) (i) Find the Fourier series of (8)

$$f(x)=\begin{cases} x^2\ if & 0\le x\le\pi \\ -x^2\ if & -\pi\le x\le 0\end{cases}$$

(ii) Using the method of seperation of variables, solve the partial differential equation $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ where $u(x,0)=6e^{-3x}$. (8)

Or

(b) (i) A tightly stretched flexible string has its ends fixed at $x=0$ and $x=l$. At time $t=0$, the string is given a shape defined by $f(x)=\mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at any time $t>0$. (8)

(ii) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 y}{\partial x^2}$ with boundary conditions $u(x,0)=3\sin n\pi x$, $u(0,t)=0$ and $u(1,t)=0$, where $0<x<1,\ t>0$. (8)