(ii) Two jointly stationary random processes $X(t)$ and $Y(t)$ have the cross power spectral density given by $S_{xy}(w) = \frac{1}{-w^2 + j4w + 4}$. Find the corresponding cross correlation function. (8)

15. (a) (i) A random process $\{X(t)\}$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t \geq 0$. If the autocorrelation function of the process is $R_{xx}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $Y(t)$. (10)

(ii) A linear system has a transfer function given by $H(w) = \frac{w}{w^2 + 15w + 50}$. Determine the power spectral density of the output when the input function is white noise that has a mean square value of $1.2V^2/Hz$. (6)

Or

(b) $X(t)$ is a Wide-Sense stationary process that is the input to a linear system with the transfer function $H(w) = \frac{1}{a + jw}$ where $a > 0$. If $X(t)$ is a zero-mean white noise with power spectral density $\frac{N_0}{2}$, determine the following

(i) The impulse response $h(t)$ of the system

(ii) The cross-power spectral density $S_{XY}(w)$ of the input process and the output process $Y(t)$

(iii) The cross correlation function $R_{YX}(\tau)$ of $Y(t)$ and $X(t)$

(iv) The power spectral density $S_{YY}(w)$ of the output process. (4 × 4 = 16)

Reg. No. :

Question Paper Code : 80219

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Computer and Communication Engineering

MA 8451 — PROBABILITY AND RANDOM PROCESSES

(Common to Electronics and Communication Engineering/ Electronics and Telecommunication Engineering)

(Regulation 2017)

Time : Three hours Maximum : 100 marks

(Use of statistical table is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that for any events A and B in S, $P(B) = P(B/A)P(A) + P(B/\overline{A}) \cdot P(\overline{A})$.

2. Find the second moment about the origin of the Geometric distribution with parameter p.

3. The joint pdf of a bivariate random variable (X, Y) is given by $f_{xy}(x, y) = \begin{cases} k, & 0 < y \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$ where k is a constant. Determine the value of k.

4. Define covariance and coefficient of correlation between two random variables x and y.

5. Define Markov process.

6. Examine whether the Poisson process $\{X(t)\}$, given by the probability law $p\{X(t) = r\} = \frac{e^{-\lambda t}(\lambda t)^r}{r!}$, $r = 0, 1, 2.....$ is covariance stationary.

7. State any two properties of auto correlation function.

8. Show that the power spectrum of a (real) random process $\{X(E)\}$ is real.

9. State fundamental theorem on the power spectrum of the output of a linear system.

10. Find the system transfer function, if a linear time invariant system has an impulse function $H(t) = \begin{cases} \frac{1}{2c}, & |t| \le c \\ 0, & \text{otherwise} \end{cases}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) State and prove Baye's theorem. (8)

(ii) Derive the moment generating function of normal distribution. (8)

Or

(b) (i) Derive the moment generation function of Poisson distribution and hence find its first three central moments. (8)

(ii) Out of 800 families with 4 children each, how many families would be expected to have (8)

(1) 2 boys and 2 girls

(2) atleast one boy

(3) atmost two girls

(4) children of both sexes.

Assume equal probabilities for boys and girls.

12. (a) (i) Determine if random variables X and Y are independent when their joint PDF is given by

$$f_{xy}(x, y) = \begin{cases} Le^{-(x+y)}, & 0 \le x \le y, 0 \le y < \infty \\ 0, & \text{otherwise} \end{cases}. \quad (8)$$

(ii) Assume that the random variable S is the sum of 48 independent experimental values of the random variable X whose PDF is given by

$$f_x(x) = \begin{cases} \frac{1}{3}, & 1 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that S lies in the range (108, 126). (8)

Or

(b) Two independent variables X and Y are define by $f(x) = 4ax$, $0 \le x \le r$, $f(y) = 4by$, $0 \le y \le s$ show that $r(U, V) = \frac{b-a}{b+a}$ where $U = X + Y$ and $V = X - Y$. (16)

13. (a) (i) The random process $\{X(t)\}$ is defind as $X(t) = 2e^{-At}\sin(wt + B)u(t)$ where $u(t)$ is the unit step function and the random variables A and B are independent, A is uniformly distributed in (0, 2) and B is uniformly distributed in $(-\pi, \pi)$. Verify whether the process is wide sense stationary. (10)

(ii) Prove that the inter arrival time of a poisson process with parameter λ has an exponential distribution with mean $\frac{1}{\lambda}$. (6)

Or

(b) (i) Find the nature of the states of the Markov Chain with three states 0, 1, 2 and with one step transition probability matrix, $P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$. (8)

(ii) If $\{X_1(t)\}$ and $\{X_2(t)\}$ represent two independent poisson processes with parameters $\lambda_1 t$ and $\lambda_2 t$ respectively, then prove that $P[X_1(t) = X / X_1(t) + X_2(t) = n]$ is Binomial with parameters n and p where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$. (8)

14. (a) (i) A stationary random process $\{X(t)\}$ has the power spectral density $S_{xx}(w) = \frac{24}{w^2 + 16}$. Find the mean-square value of the process by Brute-Force method. (10)

(ii) Prove that autocorrelation function of the random process with the power spectral density given by $S_{xx}(w) = \begin{cases} s_0, & |w| < w_0 \\ 0, & \text{otherwise} \end{cases}$ is $\frac{s_0}{\pi\tau}\sin\omega_0\tau$. (6)

Or

(b) (i) Two random processes $X(t)$ and $Y(t)$ are defined as follows.

$X(t) = A\cos(wt + \theta)$

$Y(t) = B\sin(wt + \theta)$

where A, B and w are constant and θ is a random variable that is uniformly distributed between 0 and 2π. Find the cross correlation function of $X(t)$ and $Y(t)$ and show that $X(t)$ and $Y(t)$ are jointly WSS. (8)