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M.E. DEGREE EXAMINATION, JUNE 2012.

First Semester

Applied Electronics/M.E. VLSI Design/M.E. Medical Electronics

MA 9217/281108 — APPLIED MATHEMATICS FOR ELECTRONICS ENGINEERS

(Common to M.E. VLSI Design and M.E. Medical Electronics)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Give an example from day - to - day life of each type of fuzzy proposition.

2. Suppose we have a fuzzy conditional and qualified proposition,

   p: If X is A then Y is B is very true

   where $A = \frac{1}{x_1} + \frac{0.5}{x_2} + \frac{0.7}{x_3}$, $B = \frac{6}{y_1} + \frac{1}{y_2}$, and S stands for very true : let $S(a) = a^2$ for all $a \in [0, 1]$. Given a fact "X is A'", where $A' = \frac{0.9}{x_1} + \frac{0.6}{x_2} + \frac{0.7}{x_3}$, we conclude that "$Y$ is B'". Calculate B'.

3. Under what conditions Cholesky decomposition method is used?

4. Give two applications on Toeplitz matrices.

5. In good years, storms occur according to a Poisson process with rate 3 per unit time, while in other years they occur according to a Poisson process with rate 5 per unit time. Suppose next year will be a good year with probability 0.3. Let $N(t)$ denote the number of storms during the first t time units of next year. Is $N(t)$ a Poisson Process? Does $N(t)$ have stationary increments? Why or why not?

6. Suppose that people immigrate into a territory at an average rate 1 per day. What is the expected time until the tenth immigrant arrives? What is the probability that the elapsed time is between the tenth and the eleventh arrival exceeds two days?

7. Write Principle of optimality.

8. Give a short note on three basic elements in the Dynamic programming model.

9. Give Little's law and its physical meaning in queueing theory.

10. What do you mean steady state analysis of queueing model?

PART B — (5 × 16 = 80 marks)

11. (a) Consider a fuzzy logic based on the standard logic operators (min, max, 1-a). For any two arbitrary propositions, A and B, in the logic, assume that we require that the equality $\overline{A \wedge \overline{B}} = B \vee (\overline{A} \wedge \overline{B})$ holds. Imposing such a requirement means that pairs of truth values of A and B become restricted to subset of $[0,1]^2$. Show exactly how they are restricted.

Or

(b) Suppose there are five people in a women's figure skating competition. They are Anny, Bonnie, Cathy, Diana and Eve. Assume that their relative goodness of performance is given by a fuzzy set $E = \frac{1}{Anny} + \frac{0.9}{Bonnie} + \frac{0.5}{Cathy} + \frac{0.9}{Diana} + \frac{0.1}{Eve}$. Using the fuzzy quantifiers of first and second kind, determine the truth values of the following fuzzy propositions :

(i) There are about three persons who had good performances

(ii) Most of them have good performance

(iii) About half of them have good performance.

12. (a) (i) Find the QR — decomposition of the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 7 \\ 0 & -1 & -1 \end{bmatrix}$

(ii) Prove if Q is an $n \times m$ matrix with n ≥ m then the columns of Q are an orthonormal set of vectors in R^n with the standard Euclidean inner product if and only if $Q^T q = I_m$.

Or

(b) (i) Find the least squares solution to the following system of equations

$$\begin{pmatrix} 2 & -1 & 1 \\ 1 & -5 & 2 \\ -3 & 1 & -4 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 5 \\ -1 \end{pmatrix}$$

(ii) Suppose that A has linearly independent columns. Then the normal system associated with $Ax = b$ can be written as, $Rx = Q^T b$.

13. (a) (i) A father asks his sons to cut their backyard lawn. Since he does not specify which of the three sons is to do the job, each boy tosses a coin to determine the odd person, who must then cut the lawn. In the case that all three get heads or tails they continue tossing until they reach a decision. Let p be the probability of heads and $q = 1 - p$, the probability of tails, Find the probability that they reach a decision in less than n tosses.

(ii) Suppose that, on average, the number of β – particles emitted from a radioactive substance is four every second. What is the probability that it takes at least 2 seconds before the next two β – particles are emitted?

Or

(b) (i) Suppose that lifetimes of light bulbs produced by a certain company are normal random variables with mean 1000 hours and standard deviation 100 hours. Suppose that lifetimes of light bulbs produced by a second company are normal random variables with mean 900 hours and standard deviation 150 hours. Howard buys one light bulb manufactured by the first company and one by the second company. What is the probability that at least one of them lasts 980 or more hours?

(ii) Suppose that, at an Italian restaurant, the time, in minutes. between two customers ordering pizza is exponential with parameter λ. What is the probability that the next pizza order is placed in at least t minutes but no later than s minutes ($t < s$)?

14. (a) (i) For the network in Fig. 1, it is desired to determine the shortest route between cities 1 to 7. Define the stages and the states using backward recursion, and then solve the problem.

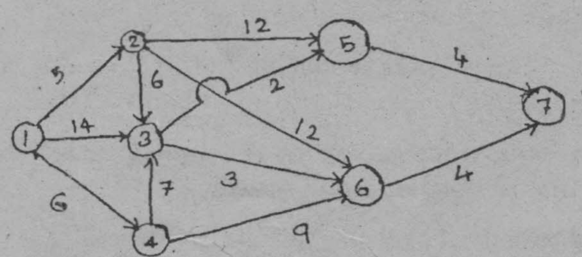

Fig. 1

(ii) A 4-ton vessel can be loaded with one or more of 3 times. The following table gives the unit weight w_i in tons and the unit revenue in thousands of dollars r_i for item i. How should vessel be loaded to maximize the total return?

Item i	w_i	r_i
1	2	31
2	3	47
3	1	14

Or

(b) A farmer owns k — sheep. At the end of each year, a decision is made as to how many to sell or keep. The profit from selling a sheep in year i is P_i. The sheep kept in year I will double in a number in year $i+1$. The farmer plans to sell out completely at the end of n - years.

(i) Derive the general recursive equation for the problem.

(ii) Solve the problem for $n = 3$ years, $k = 2$ sheep

$P_1 = \$100$ $P_2 = \$130$ and $P_3 = \$120$

15. (a) Visitors' parking at a college is limited to five spaces only. Cars making use of this space arrive according to a Poisson distribution at the rate of six cars per hour. Parking time is exponential distribution with mean of 30 mins. Visitors who cannot find an empty space on arrival may temporarily wait inside the lot until a parked car leaves. That temporary space can hold only three cars. Other cars that cannot park or find a temporary waiting space must go elsewhere. Determine the following :

(i) The probability p_n of n cars in the systems

(ii) The effective arrival rate for cars that actually use the lot

(iii) The average number of cars in the lot

(iv) The average time a car waits for a parking space inside the lot

(v) Average number of occupied parking spaces.

Or

(b) The mean rate of arrival of planes at an airport during the peak period of 20 hour, but the actual number of arrivals in any hour follows a Poisson distribution, with respective averages. When there is congestion, the planes are forced to fly over the field in the stack awaiting the landing of the other planes that arrived earlier.

(i) How many planes would be flying over the field in the stack on an average in good weather and in bad weather?

(ii) How long a plane would be in the stack and in the process of landing in good and bad weather?

(iii) How much stack and landing to allow so that priority to land out of order would have to be requested only one time in twenty?