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## Question Paper Code : 91692

M.E. DEGREE EXAMINATION, JANUARY 2012.

First Semester

Applied Electronics

MA 9217 — APPLIED MATHEMATICS FOR ELECTRONICS ENGINEERS

(Common to M.E. VLSI Design and M.E. – Medical Electronics)

(Regulation 2009)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $(P \to R) \vee ((S \to Q) \vee P)$ is a tautology without using truth tables.

2. State the two axioms of a function which is to be considered a fuzzy complement.

3. State any two properties of singular value decomposition.

4. Define band matrix. Give an example.

5. A continuous random variable $X$ has a probability density function $f(x) = k(1 + x)$, $2 < x < 5$. Find the value of $k$ and also $P(X < 4)$.

6. Given a random variable $X$ with probability density function $f_X(x) = 2x$, $0 < x < 1$, find the probability density function of $Y = 8X^3$.

7. State the principle of optimality.

8. List any four applications of dynamic programming.

9. List the characteristics and Kendal's notation of a queueing system.

10. Write down Little's formula for single server finite capacity Poisson queue.

PART B — (5 × 16 = 80 marks)

11. (a) Classify fuzzy propositions. Explain each type with examples. (16)

Or

(b) In network computing many applications involve communication between two separate systems interconnected in a network. Two metrics of interest during the interaction of these client–and–server systems are the response on the system where the user resides and the load on the remote system. Let $X$ represent the universe of response $X = \{1, 2, 3, 4, 5\}$ and $Y$, $\delta$ the universe of load $Y = \{1, 2, 3, 4\}$ where lower numbers correspond to faster response and higher load respectively. Now the two fuzzy variables $A$ the average response and $B$ the medium load where

$$A = \left\{\frac{0.2}{1} + \frac{0.5}{2} + \frac{1}{3} + \frac{0.3}{4} + \frac{0}{5}\right\} \text{ and } B = \left\{\frac{0}{1} + \frac{0.7}{2} + \frac{0.4}{3} + \frac{0.1}{4}\right\}$$

Find the implication $A \to B$ using the classical approach $R = (A \times B) \cup \left(\overline{A} \times Y\right)$ (or) $\mu_R(x, y) = \max\{[M_A(x) \wedge \mu_B(y)], (1 - \mu_A(x))\}$ what 'degree of load' would be associated with a new fuzzy set $A'$ denoting quick response? Let $A' = \left\{\frac{0.9}{1} + \frac{0.7}{2} + \frac{0.3}{3} + \frac{0.1}{4} + \frac{0}{5}\right\}$. Using max-product composition find $B' = A' \circ R$ what might this "degree of load" be called?

12. (a) (i) Find the least square solution and least square error of the equation $\begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. (6)

(ii) Invert the Toeplitz matrix $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$. (10)

Or

(b) Find the singular value decomposition of $U = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$. (16)

13. (a) (i) The probability mass function of a discrete random variable $X$ is given by $P(X = x) = \frac{1}{2^x}$, $x = 1, 2, 3, \ldots$. Find the moment generating function, mean, variance and also $P(X$ is even), $P(X \geq 5)$, and $P(X$ is divisible by 3). (10)

(ii) If a random variable $X$ follows a uniform distribution in the interval (0, 2) and $Y$ follows an exponential distribution with parameter $\lambda$, find $\lambda$ such that $P(X < 1) = P(Y < 1)$. (6)

Or

(b) (i) By finding the moment generating function of a Geometric distribution, find its mean and variance. And also prove the memoryless property of geometric distribution. (10)

(ii) A random variable $Y$ is defined by $Y = \frac{2|X| + X}{|X| 2}$, where $X$ is another random variable. Determine the density and distribution function of $Y$. (6)

14. (a) A 6-ton vessel can be loaded with one or more of three items. The following table gives the unit weight, $w_i$ in tons and the unit value in thousands of rupees, $r_i$. How should the vessel be loaded to maximize the total value? Use dynamic programming approach to solve the problem. (16)

| $i$ | 1 | 2 | 3 |
|---|---|---|---|
| $w_i$ | 4 | 2 | 1 |
| $r_i$ | 80 | 60 | 40 |

Or

(b) Solve the linear programming problem by dynamic programming approach : Max $z = 4x_1 + 14x_2$ subject to $2x_1 + 7x_2 \leq 21$, $7x_1 + 2x_2 \leq 21$ and $x_1, x_2 \geq 0$.

15. (a) (i) Consider a state automobile inspection station with three inspection stalls, each with room for only one car. It is reasonable to assume that cars wait in such a way that when a stall becomes vacant, the car at the head of the line pulls up to it. The station can accommodate at most four cars waiting at one time. The arrival pattern is Poisson with a mean of one car every minute during the peak periods. The service time is exponential with a mean of 6 minutes. Find the average number in the system during the peak periods, the average wait and the expected number per hour that cannot enter the station because of full capacity.

(ii) Cars arrive at a petrol pump, having one petrol unit in Poisson fashion with an average of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find the average number of cars in the system, average waiting time in the queue and the probability that the number of cars in the system is 2. (6)

Or

(b) (i) Find the state probabilities $\{P_n(t)\}$ for an $M / M / 1$ model and also solve the steady state difference equations for $\{P_n\}$. (10)

(ii) Customers arrive at a one-man barber shop according to Poisson process with a mean inter arrival time of 20 min. Customers spend an average of 15 min in the barber's chair. What is the expected number of customers in the barber shop and in the queue? What is the probability that more than 3 customers in the system? What is the probability that the waiting time in the system is greater than 20 min? (6)

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