Roll No. | | | | | | | | | | |

Question Paper Code : X11319

B.E / B. Tech DEGREE EXAMINATIONS APRIL / MAY 2021

Second Semester

Artificial Intelligence and Data Science

MA8252 LINEAR ALGEBRA

(Common to Computer Science and Business System)

(Regulations 2017)

Time: 3 Hours   Answer ALL Questions   Maximum: 100 Marks

PART- A (10 x 2 = 20 marks)

1. State the difference between Gauss Elimination and Gauss Jordan methods.
2. Is the system of equations $10x + y + z = 12\,;\, 2x + 10y + z = 13\,;\, x + y + 5z = 7$ consisent? Justify.
3. Determine whether the vectors $v_1 = (1, -2, 3)$, $v_2 = (5, 6, -1)$, $v_3 = (3, 2, 1)$ form a linearly dependent or linearly independent set in R^3.
4. Let $V = R^2$ and $W = \{(x, y): x + y = 5\}$. Is W a subspace of V?
5. Show that T: $R^2 \to R^2$ defined by $T(a_1,a_2) = (2a_1 + a_2, a_1)$ is linear.
6. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a2.$ Write the matrix of the linear transformation.
7. Define inner product space.
8. Let V be an Inner product space then for all $x, y \in V$ and $c \in F$, show that
$$\|cx\| = |c|\|x\|.$$
9. Explain power method to find the dominant eigen value of a matrix.
10. Find the singular values of $= \begin{pmatrix} 2 & 1 & -1 \\ -2 & -1 & 1 \end{pmatrix}$.

Part – B (5 x 16 = 80 marks)

11. a) Solve the system using Gauss elimination method (13)

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

OR

b) Solve by Gauss Jordan method (13)

$$3x + y - z = 3 \,; 2x - 8y + z = -5 \,; x - 2y + 9z = 8$$

12. a) Prove that $P_n(R)$, the set of all polynomials of degree at most n with real coefficient is a vector space under usual addition and constant multiplication of polynomial. (13)

OR

b) Let W_1 and W_2 be subspaces of a vector space V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. (13)

13. a) A linear transformation $T: R^2 \to R^3$ is defined by $T(x_1, x_2) = (x_1 + x_2,\, x_1 - x_2, x_2)$. Verify dimension theorem. (13)

OR

b) Let $A = \begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ 6 & -6 & 3 \end{bmatrix}$. Is A diagonalizable? Find an invertible matrix Q and a diagonal matrix D such that $D = Q^{-1}AQ$. (13)

14. a) Find an orthonormal basis of $\mathbb{R}^3$, given that an arbitrary basis of $\mathbb{R}^3$ is $v_1 = (1,1,1)$, $v_2 = (0,1,1)$ and $v_3 = (0,0,1)$ using Gram-Schmidt process. (13)

OR

b) Obtain the Least square approximation that fits the following data $(1,2), (2,3), (3,5)$ and $(4,7)$. (13)

15. a) Using Jacobi Rotation method find the eigen values and eigen vectors of $A = \begin{pmatrix} 6 & \sqrt{3} \\ \sqrt{3} & 4 \end{pmatrix}$. (13)

OR

b) Find the QR factorization of $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{pmatrix}$. (13)
