18/07/22 – FN.

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**Question Paper Code : 20813**

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Third/Fourth/Seventh Semester

Agriculture Engineering

MA 8391 – PROBABILITY AND STATISTICS

(Common to : Biomedical Engineering/Electrical and Electronics Engineering/Environmental Engineering/Industrial Engineering/Industrial Engineering and Management/Manufacturing Engineering/Mechanical Engineering (sandwich)/Petrochemical Engineering/Safety and Fire Engineering/Artificial Intelligence and Data Science/Bio Technology/Bio Technology and Biochemical Engineering/Chemical Engineering/Computer Science and Business Systems/Fashion Technology/Food Technology/Handloom and Textile Technology/Information Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology/Plastic Technology/Polymer Technology/Textile Chemistry/Textile Technology)

(Regulations 2017)

(Use of Statistical Table is permitted)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The probability that in a factory, a worker is skilled is 0.4. Find the probability that out of 3 workers, atleast two will be skilled.

2. If $X$ is exponentially distributed with parameter $\lambda$ find the value of $k$ such that $P(X > k) | P(X \le k) = a$.

3. If a two dimensional random variable $(X, Y)$ has the Joint probability distribution $f(x, y) = \begin{cases} \frac{kx}{y}, & x = 0, 6, 12 \text{ and } y = 1, 3, 6 \\ 0, & \text{otherwise} \end{cases}$

   Find $R$.

4. State Central Limit Theorem.

5. A random sample of 60 students gave an average weight of 58 kg with a standard deviation of 2 kg. Find 99% confidence limits of the mean of the population.

6. State any two applications of Chi-square test.

7. Write any two differences between RBD and LSD.

8. Express $2^2$ factorial designs.

9. What is control chart?

10. Define two-sided tolerance limits.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let $X$ be a random variable. Given $P(X \le x) = \frac{4x^3}{500}$ for $x = 1, 2, 3, \ldots n$

Find

(1) the value of $n$ such that the given function is a cumulative distribution function.

(2) the probability mass function of $X$

(3) the probability mass function of $X^2 + 2X$. (6)

(ii) In a survey, the mean weight of a new-born baby is observed as 10 lbs and standard deviation 1.5 lbs. If the weights are normally distributed then what value of $x$ does the interval $[10 - x, 10 + x]$ include 85% of birth weights? (5)

(iii) In a certain city, the probability that a patient recovers from typhoid is 0.5. If 16 people are known to have affected by typhoid then what is the probability that (1) atleast 2 to survive (2) atmost 3 to survive? (5)

Or



(b) (i) A continuous random variable $X$ has the pdf

$$f(x) = \begin{cases} K(x+1), & 2 < x < 5 \\ 0, & otherwise \end{cases}$$

Find,

(1) $K$

(2) $P(3 < X < 4)$

(3) $P(X > 1.5 \mid X > 1)$ (6)

(ii) Out of 1000 marbles, 80 are green and the rest black. If 70 marbles are picked at random then find the probability of selecting (1) 3 green balls (2) not more than 3 green balls in the sample. (5)

(iii) The life of a component is normally distributed with a mean of 250 hours and standard deviations $s$ hours. Find the maximum value of $s$, so that the probability of the component to have a life between 200 and 300 hours is 0.70. (5)

12. (a) (i) If the joint pdf of $(X, Y)$ is given by $f(x, y) = \frac{1}{2} x e^{-y}$, $0 < x < 2$, $y \ge 0$ then find the

(1) marginal distribution function of $X$ and $Y$

(2) $P(X < 1 \mid Y < 2)$

(3) Are X and Y independent?

(4) $E(4X)$. (8)

(ii) The covariance of X and Y of the following data is 3.85

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $X$: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $Y$: | 9 | 8 | $y_3$ | 11 | 12 | 13 | 14 |

Find $y_3$ and hence compute the correlation coefficient of $X$ and $Y$. (8)

Or

(b) (i) Let X and Y are independent RVs with means 4 and 8 and standard deviations $\sqrt{5}$ and $\sqrt{10}$ respectively. Obtain the correlation coefficient between S and T, where $S = 2X - 4Y$ and $T = 3X + 2Y$. (8)

(ii) The joint p.d.f. of $(X,Y)$ is given by $f_{XY}(x, y) = \begin{cases} e^{-x-y}, & x > 0, y > 0 \\ 0 & , elsewhere \end{cases}$.

Find the joint p.d.f. of $(U, V)$, where $U = \frac{X}{Y}$ and $V = X + Y$. Are $U$ and $V$ independent? (8)

13. (a) (i) In an investigation of fitness and diet of two groups of patients from different social status were enlisted below :

| Social status | Poor | Middle class |
|---|---|---|
| Fitness | | |
| Below normal | 130 | 20 |
| Normal | 112 | 118 |
| Above normal | 24 | 96 |

Discuss the association with the fitness and their social status. (8)

(ii) During the countrywide investigation, the incidence of COVID-19 was found to be 2%. In a company with 400 employers, 5 are reported to be affected, whereas in another with 1200 employers 10 are found to be affected. Does this indicate any significant difference at 1% level? Also, find 99% confidence limit of their proportion difference. (8)

Or

(b) (i) Fit a binomial distribution for the following data and also test the goodness of fit. (8)

| $x$ : | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f$ : | 15 | 19 | 36 | 25 | 5 |

(ii) Two different types of drugs A and B were tried on certain patients for increasing weight. 6 patients were give drug A and 7 patients were given drug B. The increase in weights (in kg) is given below :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Drug A : | 3.6 | 3.5 | 4.2 | 4.1 | 1.4 | 2.5 | |
| Drug B : | 4.5 | 3.6 | 5.5 | 6.8 | 2.7 | 3.6 | 5.0 |

Do the drugs differ significantly with regard to their effect in increasing weight? (8)



14. (a) (i) Analyze the significance difference between three types of tyres T1, T2 and T3 at 5% level of significance.

| T1 | T2 | T3 |
|---|---|---|
| 11 | 13 | 14 |
| 18 | 17 | 13 |
| 18 | 11 | 10 |
| 17 | 10 | 12 |
| 16 | 8 | 11 |

(8)

(ii) Four different drugs have been developed for a certain disease. These drugs are used in 3 different hospitals and the results given below :

| Hospital | Drug I | Drug II | Drug III | Drug IV |
|---|---|---|---|---|
| A | 10 | 14 | 19 | 20 |
| B | 11 | 15 | 17 | 21 |
| C | 9 | 12 | 16 | 19 |

Discuss the difference between (i) Drugs (ii) Hospitals. (8)

Or

(b) (i) Analyze the variance in the following Latin square. (8)

| | | | |
|---|---|---|---|
| B10 | C14 | D19 | A20 |
| A11 | D15 | C17 | B21 |
| D9 | A12 | B16 | C11 |
| C8 | B13 | A17 | D10 |

(ii) The following table gives the lives in (hours) of electric bulbs of four companies A, B and C.

| | | | | | | |
|---|---|---|---|---|---|---|
| A | 1600 | 1600 | 1700 | 1800 | 1900 | 1800 |
| B | 1500 | 1600 | 1600 | 1700 | 1800 | |
| C | 1400 | 1400 | 1500 | 1500 | 1600 | 1700 |

Do the analysis of variance and test the homogeneity of the mean lives of the bulbs of three companies. (8)

15. (a) (i) Construct a control chart for defectives for the following data :

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Sample No : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| No. inspected : | 90 | 85 | 65 | 70 | 80 | 70 | 95 | 75 |
| No. of defectives : | 9 | 7 | 4 | 3 | 2 | 8 | 6 | 5 |

Comment on the nature of process. (8)

(ii) The following data show the values of sample mean $\overline{x}$ and the range $R$ for the sample size 5 each. Calculate the values for central line and control limits for mean-chart and range chart and determine whether the process is in control.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sample No : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $\overline{X}$ : | 11.2 | 11.8 | 10.8 | 11.6 | 11 | 9.6 | 10.4 | 9.6 | 10.6 | 10 |
| $R$ : | 7 | 4 | 8 | 5 | 7 | 4 | 8 | 4 | 7 | 9 |

(Conversion factors for $n = 5$ are $A_2 = 0.577$, $D_3 = 0$ and $D_4 = 2.115$) (8)

Or

(b) (i) 12 tape-recorders were examined for quality control test. The number of defects in each tape-recorder is given below. Construct an appropriate control chart and comment on the state of control. (8)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Unit : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| No. of defects : | 2 | 4 | 3 | 1 | 1 | 2 | 5 | 3 | 6 | 7 | 3 | 1 |

(ii) Write the role and advantages of SQC. (8)

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